AN ANALYTICAL STUDY OF VIBRATION TRANSMISSION IN TYPICAL SHIPBOARD INSTALLATIONS

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ABSTRACT

The analysis of vibration generation and transmission in shipboard machinery installations had defied rigorous mathematical description. After certain simplifying assumptions have been made, the vibration response of the installation can be predicted analytically. Theeobject of this thesis is to demonstrate the method by which the simplified system can be analyzed to give vibration trends and characteristics. The method incorporates the four pole parameter technique of analysis and a systematic study of the effects of installation modifications.

The four pole parameters of a system are those expressions which inter-relate two basic input and output quantities. For vibrating mechanical systems these quantities are force and velocity. The four pole parameters must be derived or measured for the mechanical elements which transmit the force and velocity through the installation. In a shipboard installation these elements are the machine, mounting, foundation, hull, and water. After the four pole parameters are obtained for these five elements, matrix notation and combination permit a transmission study of the entire five element system. The computational assistance of a digital computer is an indispensable aid in this portion of the study.

Numerous conclusions and recommendations were obtained from the analysis and from the results of the systematic study. The most important of these were:

(1) For the purpose of vibration analysis a complex machinery installation can be represented by a relatively simplified system. This representation and the use of the four pole parameter technique are sufficient to establish trends and characteristics.

(2) The stiffness of the resilient mounting has a dominant effect on the vibration transmission at all frequencies.

(3) A Portsmouth mounting with decreased stiffness and increased damping and synthetic rubber damping applied to the foundation were the most effective installation modifications in providing overall transmission reduction.

(4) As new ideas for reducing vibration transmission are introduced, they should be investigated by means of the four pole parameter technique.

(5) A model of the approximate system of this thesis should be constructed to permit experimental verification of the four pole parameter technique.

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CREDIT

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LIST OF SYMBOLS

A	Cross-sectional area, beam or column, in^2
C	Equivalent spring compliance, in/lbf
c	Velocity of sound in water, in/sec
E	Elastic modulus, psi
F	Force, lbf
f	Frequency, cps
G	Mounting elastic modulus, psi
I	Moment of inertia, in^4
K	Equivalent spring stiffness, lbf/in
k	Equivalent spring stiffness, lbf/in
L	Length of element, in
M	Mobility, in/sec-lbf
m	Mass, lbm
p	Distance to driving point, in
Q_0	Maximum transmission at ω_0, dimensionless
R	Damping constant, lbf-sec/in
r	Damping constant, lbf-sec/in
R*	Specific resistance, lbf-sec/in^3
S	Mount cross-sectional area, in^2
T	Transmission, dimensionless
t	Element thickness, in
t	Time, sec
V	Velocity, in/sec
W	Element weight, lbf
X	Reactance, lbf-sec
x	Longitudinal distance, in
y	Transverse displacement, in

Z	Impedance, lbf-sec/in
ϵ	Four pole parameter
λ	Constant in natural frequency equation
ρ	Mass density, lbm/in^3
ϕ	Normal function
ω	Frequency, rad/sec
ω_0	Rated natural frequency of mount at rated load, rad/sec
ω_t	Frequency of first high frequency transmission peak of mount, rad/sec

CHAPTER I

INTRODUCTION

1.1 Importance of Vibration Transmission

The problem of noise control is one of major importance to naval engineers. A ship with a high noise level operates under the joint handicaps of being easily detectable by the enemy's listening devices and of reducing the effectiveness of its own sonar. Noise control implies the control of vibrations since acoustic or noise energy in a medium is synonymous with vibration transmission in that medium. A thorough understanding of the vibration transmission characteristics of shipboard installations can provide the solution of the noise control problem.

1.2 Current Developments

Spurred by the Navy's need for quieter submarines, many facilities and laboratories have been devoting time and effort towards finding a solution of the noise problem. Many studies are currently in progress seeking mechanical means of reducing vibrations in machinery installations. Much of this is being financed by Navy contracts; thus, most of the major new developments have shipboard applications.

The development receiving the most application at present in the Navy is the rubber-type resilient mount. These Navy mountings have been developed by the Engineering Experiment Station, Annapolis, Maryland and the Portsmouth Naval Shipyard, Portsmouth, New Hampshire (15). The effectiveness of resilient mountings has been studied by Mr. A. O. Sykes of the David Taylor Model Basin (13, 14). Studies have also been made of the properties of other rubber-like materials and of their effectiveness when used as mountings (17).

It is becoming more apparent to interested engineers that the other elements in the vibration transmission path from source to water must

be investigated. Work has been done in determining vibration characteristics in machinery, the most common vibration source (12). Machinery foundation vibrations and techniques of damping them are being experimentally evaluated by the Sound and Vibration Section at the Electric Boat Company (20). This work has emphasized the importance of the foundation in the vibration transmission path.

The final item in the vibration transmission path is the ship's hull. Measurements of the vibration response of submarine hulls have been made. This empirical data gives a possible means of predicting the interacting effects of foundation, hull, and water (11).

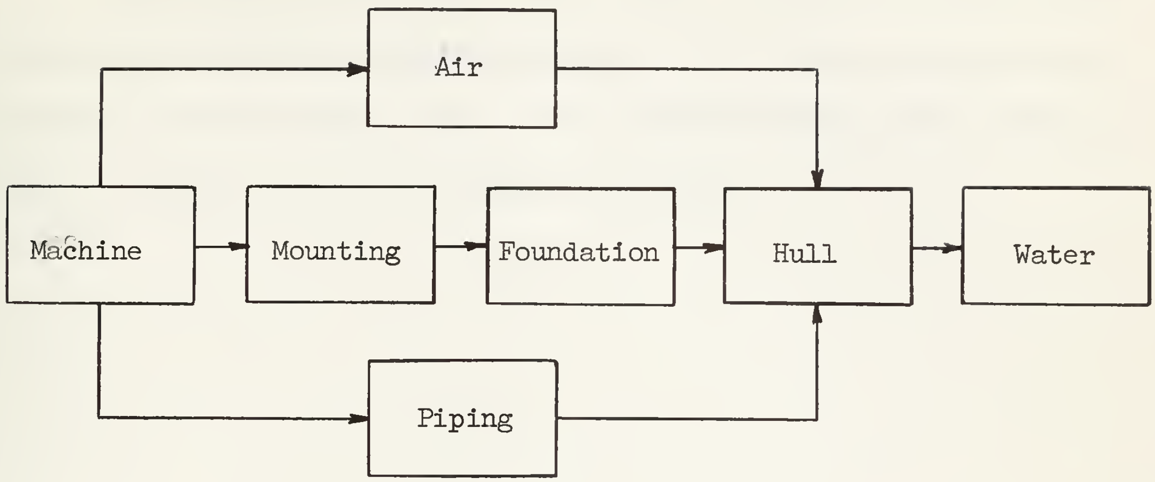
It has been recognized that the time to apply noise control measures aboard ship is in the preliminary design and construction stages. This concept has been applied to the SSN 593, Thresher, to be commissioned this summer. A consistent set of specifications, setting allowable noise and vibration levels, have been developed for this ship.

1.3 The System Under Study

There are several means by which noise or vibration aboard ship can be transmitted from its source to the water. If an operating machine is considered as a typical noise source, its vibration can reach the hull and water by several paths. These are airborne transmission, supporting structure transmission, and transmission through connected piping. The vibration paths are best illustrated by the component block diagram in Figure I.

Figure I

System Block Diagram



The initial step in formulating an analytic approach to this transmission problem is to assume that each of these paths can be treated separately. This thesis is limited to an analytic study of the transmission through the supporting structure. The path under study, as indicated in Figure I, consists of machine, mounting, foundation, hull, and water.

1.4 The Analytic Approach

The exact describing equations of the transmission phenomena through the above mentioned mechanical elements would be an ideal analytic tool for the investigation of this problem. The actual system, however, defies any clear-cut description, and to obtain the describing equations is a Herculean task. The system is non-linear and possesses many degrees of freedom. The vibration of most machines is of a random nature. In addition, these machines are mounted on complex structures.

To circumvent the above complexities, simplifying assumptions must be made to obtain a workable mathematical model. These assumptions are completely enumerated in Chapter II of this thesis. The simplified

model is not intended to give the complete picture of vibration transmission in the actual installation.

The object of this thesis is to make a systematic study of the analytically-described, simplified system. It is hoped that from this simplified study vibration trends and preferred design practice for the complex machinery system will become available.

CHAPTER II

PROCEDURES

2.1 The Preliminary Experiment

The authors first became interested in an analytic approach to vibration transmission in shipboard installations while on temporary duty at the Admiralty Engineering Laboratory, West Drayton, England. A series of experiments were being conducted there to determine isolation characteristics of Admiralty resilient mountings. Results had been obtained from one of these basic experiments, and the authors investigated various approaches to the mathematical verification of this data.

The experimental results studied were obtained by driving a beam supported by an Admiralty mounting with a typical vibration generator. This experiment is described in Appendix A-1.

Most previous experiments investigating the characteristics of mountings treated the supported member as a lumped mass. This treatment facilitated use of a theory involving electrical analogies (14). In this case the beam was used as the supported member because of its non-rigidity.

A recent paper by R. D. Cavanaugh and J. E. Ruzicka provided a method by which a vibrating beam-spring system could be analyzed (4). This method was the four pole parameter technique. The four pole parameter method had been widely used to analyze mechanical systems; therefore, it was selected as the technique to provide the mathematical verification of the beam-mounting experiment.

2.2 The Four Pole Parameter Technique

2.2.1 General

There are many systems in nature which have two basic quantities as input and two basic quantities as output. An electric circuit element is one of these systems. In this case voltage and current are both input

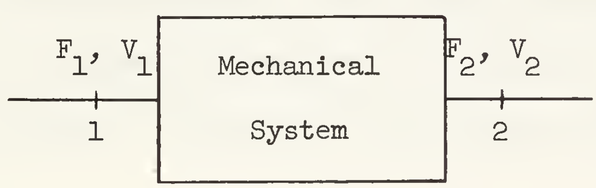
[illegible]

and output. All these systems are called "four poles" since four quantities become inter-related by some action within the system.

Electrical engineers have devised a technique of analysis for circuit elements called the four pole parameter technique. Each circuit element is considered independently, and complex circuitry is developed by connecting the various elements. Recently the four pole parameter technique has been applied to mechanical vibration problems where the mechanical element considered has a force and velocity as both input and output.

A typical mechanical system can be represented in block diagram form as illustrated in Figure II.

Figure II



F_1 and V_1 are the input force and velocity, while F_2 and V_2 are the output force and velocity. This system can be described in general by the following equations:

$$F_1 = \epsilon_{11} F_2 + \epsilon_{12} V_2 \tag{1}$$

$$V_1 = \epsilon_{21} F_2 + \epsilon_{22} V_2 \tag{2}$$

ϵ_{11}, ϵ_{12}, ϵ_{21}, and ϵ_{22} are the four pole parameters of the mechanical system.

Using matrix notation, equations (1) and (2) become:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} . \tag{3}$$

The determination of the four pole parameters comprises the major portion of the technique. Various rules must also be developed to permit combination of the matrices to represent mechanical elements in various multiple configurations.

2.2.2 Development of the Four Pole Parameters

Basically the solution for the four pole parameters of the mechanical system can be reduced to the following procedure:

1. Investigate the system and obtain its performance equations.
2. Solve the equations imposing the proper boundary conditions. If non-linearities exist in the system it may be necessary to apply linearization techniques at this step of the procedure.
3. Reduce the solutions to the form of equations (1) and (2). The four pole parameters can then be written by direct comparison.

This procedure is applied to several basic mechanical elements in Appendix A-2.

The above procedure can be specifically applied to the mechanical systems found in the analysis of vibration transmission. Equations (1) and (2) can be solved for the four pole parameters by imposing the condition that the output, which is station 2 in Figure II, is either blocked or free. The blocked condition implied the velocity at station 2 is zero; the free condition implies the force at station 2 is zero.

$$\epsilon_{11} = \frac{F_1}{F_2} \quad \text{with station 2 blocked} \tag{4}$$

$$\epsilon_{12} = \frac{F_1}{V_2} \quad \text{with station 2 free} \tag{5}$$

$$\epsilon_{21} = \frac{V_1}{F_2} \quad \text{with station 2 blocked} \tag{6}$$

$$\epsilon_{22} = \frac{V_1}{V_2} \quad \text{with station 2 free} \tag{7}$$

The following definitions can be applied to equations (4) through (7) to simplify the notation.

$$(T_F)^b = \text{Force Transmission} = \frac{F_2}{F_1} \quad \text{with station 2 blocked}$$

$$(T_V)^f = \text{Velocity Transmission} = \frac{V_2}{V_1} \quad \text{with station 2 free}$$

$$(M_{12})^f = \text{Transfer Mobility} = \frac{V_2}{F_1} \quad \text{with station 2 free}$$

$$(M_{21})^b = \text{Transfer Mobility} = \frac{V_1}{F_2} \quad \text{with station 2 blocked}$$

Equations (4) through (7) become:

$$\epsilon_{11} = \left(\frac{1}{T_F}\right)^b \tag{4a}$$

$$\epsilon_{12} = \left(\frac{1}{M_{12}}\right)^f \tag{5a}$$

$$\epsilon_{21} = (M_{21})^b \tag{6a}$$

$$\epsilon_{22} = \left(\frac{1}{T_V}\right)^f \tag{7a}$$

Equation (3) can now be written:

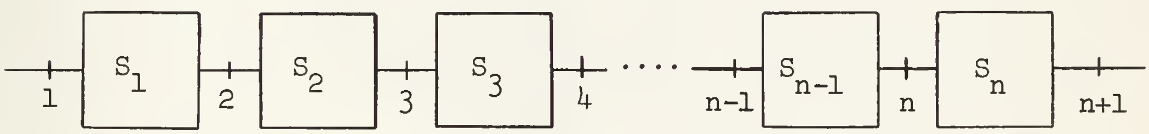
$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{T_F}\right)^b & \left(\frac{1}{M_{12}}\right)^f \\ (M_{21})^b & \left(\frac{1}{T_V}\right)^f \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{3a}$$

This development provides a convenient notation by which any system may be analyzed.

2.2.3 Combination of Four Pole Matrices

Multiple mechanical elements for which the four pole matrices have been derived can be connected in series. This is illustrated in Figure III.

Figure III



If this is the case, the overall system of n subsystems S_1 through S_n can be represented by a single four pole matrix $[S_s]$.

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} S_s \end{bmatrix} \begin{bmatrix} F_{n+1} \\ V_{n+1} \end{bmatrix} \tag{8}$$

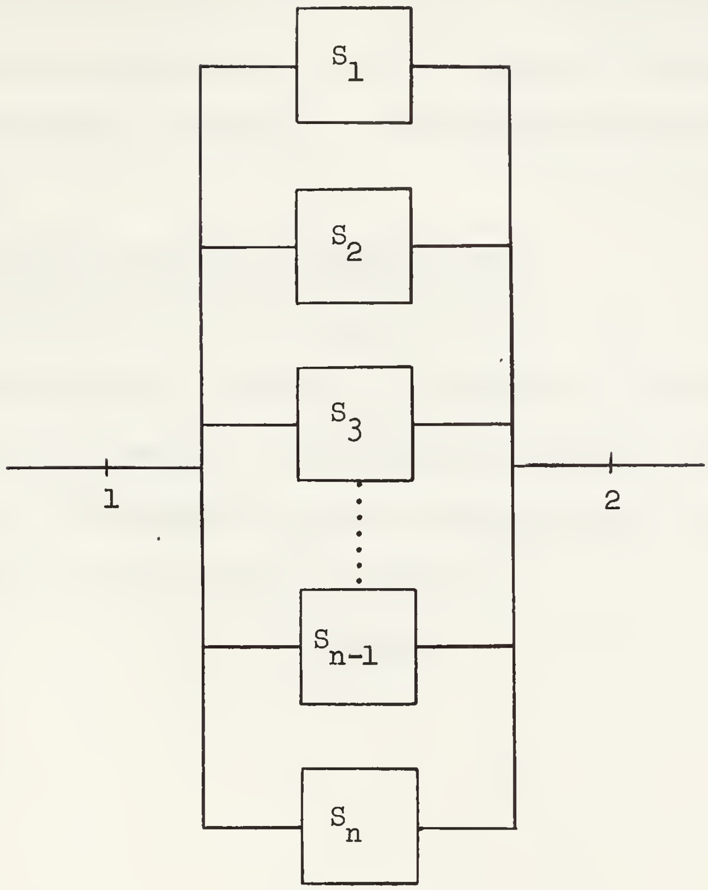
The single four pole matrix $[S_s]$ is defined:

$$[S_s] = [S_1] \times [S_2] \times [S_3] \times \ldots\ldots\ldots\ldots [S_n - 1] \times [S_n] \tag{9}$$

Multiple elements can also be connected in parallel as illustrated

in Figure IV.

Figure IV



These n parallel subsystems can be represented by a single four pole matrix $[S_p]$.

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} S_p \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix}$$

The derivation of the matrix $[S_p]$ is more complicated for the parallel than the series case. The solution for the four poles of $[S_p]$ is included in Appendix A-3.

2.2.4 Application of Four Pole Parameter Technique to Preliminary Experiment

From the preliminary experiment mentioned in section 2.1 and fully described in Appendix A-1, experimental results were available to test the

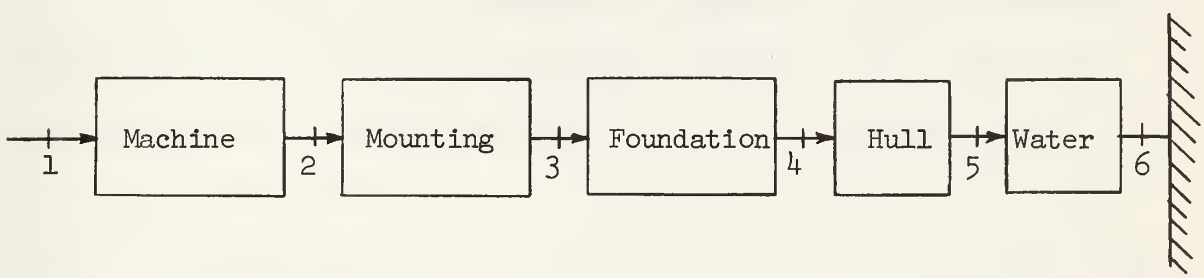
validity of the four pole parameter technique. Using this technique the isolation, which is the ratio of input force to output force for the system, was theoretically obtained as a function of driving frequency. The experimental results closely agreed with the values of isolation determined by the four pole parameter technique. This demonstrated the effectiveness of the technique.

2.3 <u>The General System and Analytic Study</u>

2.3.1 Description of the General System

As mentioned in Chapter I, the overall system which is the subject of this thesis consists of the mechnaical elements in the structural path of vibration transmission from the machine source to the water. The block diagram of this system is Figure V.

Figure V



2.3.2 The Analytic Study

The initial step in any analytic study is the enumeration of the simplifying assumptions. In this thesis these assumptions are:

1. The transmission paths in Figure I can be considered separately.

2. The principal contributors to structural vibration transmission are the force and velocity normal to the hull. All others are of

lesser importance. (This assumption has been corroborated by other investigators (17).)

3. The mechanical elements are elastic and linear.

4. The vibrations in the system are periodic.

Other assumptions have been made when analyzing the individual elements. These will be discussed in deriving the four poles of each mechanical element.

It is recognized that the above assumptions will not permit an exact solution of the vibration transmission characteristics of an actual machinery installation. As stated in Chapter I, however, the purpose of this thesis is not to accurately predict vibration transmission for any particular mechanical system. Rather it is to provide a systematic study of a hypothetical installation such that trends caused by parameter variations can be predicted. A knowledge of these trends could indicate design procedures to reduce vibration transmission.

With the use of the simplifying assumptions and the equations of dynamic analysis, the four pole parameter technique can be applied to the general system described in Section 2.3.1. The dynamic analysis relates input force and velocity to output force and velocity for each element. The principal task is the determination of the four poles which appear in these equations.

2.4 The Four Pole Parameters of the General System

2.4.1 The Machine

The problem of mathematically representing the vibration characteristics of an operating machine is a very difficult task. A study to evaluate the internal impedance of a typical machine has been made by the A. D. Little Co. (12). In this report an ideal machine is considered to possess many resonances and a complex response spectrum. Even though in

some cases the exciting internal forces are random, many machines rotate and thus exhibit a form of periodicity in their force generation. For this reason a harmonic force generator is considered as the vibration source in the A. D. Little report. In their mathematical model a harmonic force generator drives a multi-mass-spring system in which the relative spring stiffnesses and mass sizes determine the resonances. Some investigators have concluded that this assumed lumped parameter behavior is valid at frequencies of only 150 cps or less (17).

Recognizing the limitations of the lumped parameter treatment, this thesis postulates that the machine can be represented by a simple beam driven by a harmonic force generator. A simple beam by virtue of its structural properties possesses distributed parameters and eliminates the limitations imposed by the lumped parameter analysis.

The four pole parameters for the beam are developed in Appendix A-4.1.

2.4.2 The Mounting

Of the mechanical systems in the structural transmission path, the mounting is the one best approximated by theory. Historically the installation of rubber-type resilient mountings was the first remedial step taken to reduce vibrations. Consequently, more mounting theories have been developed and tested than theories for the other mechanical elements.

The theories used to develop the four pole parameters of the mounting in the general system are those proposed by D. V. Wright and A. C. Hagg (17, 7), A. O. Sykes (13, 14), and J. C. Snowden (18).

The first mounting considered in the general system is the standard Portsmouth mount, which is found in many shipboard installations today. The describing four poles for the mount are derived in Appendix A-4.2

according to the theory developed by Wright and Hagg.

The second mounting considered in the general system is one constructed of a resilient rubber-type material similar to that bearing the trade name of Thiokol R. D. The four pole parameters for this mount are developed following the concepts of Sykes and Snowden in Appendix A-4.2.

2.4.3 The Foundation

In most practical installations, machinery foundation structures are combinations of simple beams. This fact naturally suggests the use of beam vibration analysis to describe the foundation element. This analysis has been used by a number of investigators as a basic step in the description of the dynamics of foundation vibration (17, 20).

For the purpose of this investigation the foundation was simplified to be a beam resting on two columns. The describing four poles for this mechanical system are derived in Appendix A-4.3.

2.4.4 The Hull

The hull of a ship is a complex structure. Unlike other elements in the structural vibration transmission path, it cannot be represented by any single mechanical element or simple combination of elements. Experimental results are necessary to describe the response of typical hull sections (7, 11, 17, 20).

A mathematical model devised by D. V. Wright has provided a reasonable approximation of the vibration characteristics of a ship's hull. This model is used to obtain the four poles of the hull section in Appendix A-4.4.

2.4.5 The Water

The water is considered only in its effect as an attenuation element. Any entrained mass effect is included as part of the hull. The

water presents an effective specific resistance to the propagation of logitudinal pressure waves. This specific resistance is dependent on the density of sea water and the velocity of sound in that medium.

The describing four poles are derived in Appendix A-4.5.

2.5 Solution of the Problem

2.5.1 General

The derivation of an expression for vibration transmission through the mechanical elements of the general system is the key to achieving the purpose of this thesis. The solution of this problem involves the combination of the five basic subsystems described in Section 2.4 into a single complex system. The four pole parameters of the combined system are then obtained according to the equations of combination explained in Section 2.2.3. The vibration transmission is defined as a force ratio. The force transmitted into the water is considered the significant noise producing force in the system. Therefore, the vibration transmission is defined as the ratio of the force into the water to the input force.

Since many of the four poles in the system are functions of frequency, the final ratio will be a function of frequency. Evaluation of this ratio at sufficient frequencies will give the desired transmission versus frequency data. To conform to convention transmission is converted to decibels.

The combination of the general system fourpoles and a sample calculation at one value of frequency are included in Appendix A-6.

2.5.2 Computer Application

This thesis could not have been completed without the computational assistance of the IBM 709 digital computer. The solution for vibration transmission at each frequency required a calculation of length comparable to that of Appendix A-6. This study required thousands of such calcula-

tions. The matrix notation of the four pole parameter technique was easily adapted to digital computer methods.

A computer program was written to solve the combination of five four pole matrices. Provisions were made to change any of the mechanical elements in the system of Figure V. A complete sequence of this computer program logic is explained in Appendix A-7. A most convenient feature of the program was the use of the cathode ray tube oscilloscope to graphically present the results. The scope photograph eliminated the necessity of manual plotting.

2.6 Systematic Study to Determine Trends

With the selection of the general system of Figure V and the mechanical elements in Section 2.4 to represent this system, the problem to be considered was established. The four pole parameter technique described in Section 2.2 provided an analytical means to study the problem. The final phase of the study, however, was the most important. It was to devise a consistent, systematic method of changing the general system such that many different cases might be evaluated and compared. This comparison and evaluation formed the basis for determining the results, conclusions, and recommendations of this thesis.

A detailed description of the systematic changes to the general system is given in Appendix A-5.

CHAPTER III

RESULTS

3.1 The Photo Plots

The results of the analytic study of vibration transmission in shipboard installations are presented in the form of forty-one photo plots. These transmission versus frequency plots are included in Appendix B with one plot for each of the cases mentioned in Appendix A-5.7.

3.2 Statements of Results

The following statements of results are deduced from a study of the photo plots. To preserve the systematic procedure of the study, each case is examined separately. For detailed descriptions of the systems studied, refer to Appendix A-5.

Case 1. This was the basic case involving System 1. It was selected as the basis for comparison since the elements comprising System 1 were typical of those presently found in shipboard installations.

Case 2. The System was the same as Case 1, except for the following:

a. Mount damping increased.

The first transmission peak was lowered.

Case 3. The System was the same as Case 1, except for the following:

a. Mount stiffness increased.

The transmission over the frequency range studied was increased.

Case 4. The System was the same as Case 1, except for the following:

a. Mount stiffness decreased.

The transmission over the frequency range studied was lowered by approximately 10 db.

Case 5. The System was the same as Case 1, except for the

following:

a. Thiokol R. D. mount replaced Portsmouth mount.

The first transmission peak was lowered while the second peak was removed.

Case 6. The System was the same as Case 1, except for the following:

a. Hull damping was doubled.

There was no change from Case 1.

Case 7. The System was the same as Case 1, except for the following:

a. Mount damping increased.

b. Mount stiffness decreased.

The results were the same as Case 4.

Case 8. The System was the same as Case 1, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

b. Hull damping was doubled.

The results were the same as Case 5.

Case 9. The System was the same as Case 1, except for the following:

a. Mount damping increased.

b. Mount stiffness decreased.

c. Hull damping increased twenty times.

The transmission was two or three decibels below that of Case 7 at the lower frequencies.

Case 10. The System was the same as Case 1, except for the following:

a. Viscous foundation damping was added.

The initial transmission slope was changed. The transmission was lowered at frequencies greater than 1700 cps.

Case 11. The System was the same as Case 1, except for the following:

a. Thiokol R. D. and Portsmouth mounts in parallel.

The transmission was increased at frequencies greater than 1000 cps.

Case 12. The System was the same as Case 1, except for the following:

a. Water resistance varied with frequency.

The transmission was lowered at frequencies greater than 1700 cps.

Case 13. The System was the same as Case 1, except for the following:

a. Viscous foundation damping was added.

b. Mount stiffness decreased.

The first transmission peak was shifted from 21 cps to 11 cps. Transmission was decreased over the entire frequency range by at least 10 db.

Case 14. The System was the same as Case 1, except for the following:

a. Thiokol R. D. foundation damping was added.

Transmission was lowered appreciably at frequencies above 800 cps.

Case 15. The System was the same as Case 1, except for the following:

a. Thiokol R. D. and Portsmouth mounts in parallel.

b. Portsmouth mount stiffness decreased.

The results were the same as Case 4 to 500 cps. Above 500 cps, the results were the same as Case 11.

Case 16. The System was the same as Case 1, except for the following:

a. Hull damping increased, maximum.

The transmission was reduced over the entire frequency range.

Case 17. The System was the same as Case 1, except for the

following:

a. Thiokol R. D. mount replaced Portsmouth mount.

b. Thiokol R. D. foundation damping was added.

The first and second transmission peaks were removed. The transmission was reduced at frequencies above 800 cps.

Case 18. The System was the same as Case 1, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

The vibration transmission was reduced 10 db. in the 10 to 800 cps range. Above 800 cps, the transmission was further decreased.

Case 19. The System was the same as Case 1, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

c. Mount damping increased.

The results were the same as Case 18, except the first peak was removed.

Case 20. This was the basic case involving System 2. Foundation mass increase was the only change from System 1. The transmission was lowered at frequencies from 10 to 1000 cps. The second and third transmission peaks were split.

Case 21. The System was the same as Case 20, except for the following:

a. Mount stiffness decreased.

The transmission over the frequency range studied was lowered by approximately 10 db.

Case 22. The System was the same as Case 20, except for the

following:

a. Thiokol R. D. mount replaced Portsmouth mount.

The first transmission peak was lowered; the second split peak was removed; and half of the third split peak was removed.

Case 23. The System was the same as Case 20, except for the following:

a. Mount damping increased.

b. Mount stiffness decreased.

c. Hull damping increased twenty times.

The first transmission peak was shifted below 10 cps, and transmission over the entire frequency range was lowered by 10 db.

Case 24. The System was the same as Case 20, except for the following:

a. Viscous foundation damping was added.

The split transmission peak at 340 cps was lowered, and the transmission from 1000 to 10,000 cps was lowered by 5 to 10 db.

Case 25. The System was the same as Case 20, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

b. Thiokol R. D. foundation damping was added.

The first transmission peak was removed while the second and third peaks were lowered. The transmission was considerably reduced at frequencies above 300 cps.

Case 26. The System was the same as Case 20, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

The first transmission peak was shifted below 10 cps. With the

exception of the second peak, the vibration transmission was considerably reduced over the entire frequency range.

Case 30. This was the basic case involving System 3. Foundation mass increase was the only change from System 2. The transmission was considerably lowered over the entire range except for peaks at 800 and 4000 cps.

Case 31. The System was the same as Case 30, except for the following:

a. Mount stiffness decreased.

The first transmission peak was removed, and the transmission reduced by 5 to 10 db. over the frequency range.

Case 32. The System was the same as Case 30, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

The first, second, and third transmission peaks were removed.

Case 33. The System was the same as Case 30, except for the following:

a. Mount damping increased.

b. Mount stiffness decreased.

c. Hull damping increased twenty times.

The results were the same as Case 31.

Case 34. The System was the same as Case 30, except for the following:

a. Viscous foundation damping was added.

The first transmission peak was split; the peaks at 800 and 2500 cps were lowered. The peak at 4000 cps was removed.

Case 35. The System was the same as Case 30, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

b. Thiokol R. D. foundation damping was added.

With the exception of the transmission peak at 200 cps, the vibration transmission was considerably lowered over the range of frequencies.

Case 36. The System was the same as Case 30, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

The first transmission peak was removed, and the transmission reduced by 5 to 10 db. up to 500 cps. Above 500 cps, the transmission was further reduced.

Case 50. This was the basic case involving System 4. The machine and foundation masses were ten times the values in System 1. This System was used as the basis of comparison for larger machinery installations.

Case 51. The System was the same as Case 50, except for the following:

a. Mount stiffness decreased.

The first transmission peak was shifted and lowered. The transmission was reduced by 5 to 10 db. over the

Case 52. The System was the same as Case 50, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

The first transmission peak was removed, and the transmission was lowered by 5 to 10 db. over the entire range.

Case 53. The System was the same as Case 50, except for the following:

a. Mount damping increased.

b. Mount stiffness decreased.

c. Hull damping increased twenty times.

The first transmission peak was considerably lowered. The transmission was lowered over the entire range by approximately 10 db.

Case 54. The System was the same as Case 50, except for the following:

a. Viscous foundation damping was added.

The first transmission peak was lowered; the third peak was eliminated. The transmission was lowered by 5 to 10 db. from 1000 to 10,000 cps.

Case 55. The System was the same as Case 50, except for the following:

a. Thiokol R. D. mount replaced Portsmouth mount.

b. Thiokol R. D. foundation damping was added.

The first transmission peak was removed, and the transmission was reduced by 10 to 15 db. to 300 cps. Above 300 cps, it was reduced by 20 to 40 db.

Case 56. The System was the same as Case 50, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

The first transmission peak was lowered and shifted to 12 cps. The second peak was reduced by 35 db. At higher frequencies the transmission was extremely low.

Case 57. The System was the same as Case 50, except for the following:

a. Thiokol R. D. foundation damping was added.

b. Mount stiffness decreased.

c. Mount damping increased.

The first transmission peak was removed. The second peak was reduced by 35 db. At higher frequencies the transmission was extremely low.

CHAPTER IV

DISCUSSION OF RESULTS

4.1 Preliminary Discussion

The most important single result of this thesis is the technique of analytic description of a complex shipboard machinery installation. This technique formulates a workable analytic tool for the study of vibration transmission characteristics of these complex systems. In essence this technique is the four pole parameter method. It is recommended that this method of analysis be considered in future design investigations.

4.2 Systematic Changes

4.2.1 Mounting

The properties of the resilient mounting had a profound effect on the vibration transmission. A study of the photo plots showed that stiffness in the Portsmouth mounting had a dominant iffect in the cases in which it was altered. When stiffness was increased, the transmission at all frequencies was increased; when stiffness was decreased, the transmission at all frequencies was lowered by 10 db. It is recommended that the design engineer make stiffness as low as possible and yet consistent with shock, alignment, and other factors influencing mount design.

Damping in the Portsmouth mounting was not nearly as influential as mount stiffness. However, increased damping was effective in decreasing the transmission peaks at lower frequencies. It is recommended that an effort be made to increase the damping of the Portsmouth mount by increasing the material damping or by introducing an external damping device in parallel with the mount.

The replacement of the Portsmouth mounting by a synthetic rubber

[illegible]

mounting of Thiokol R. D. material showed interesting results. The Thiokol R. D. mount lowered the first transmission peak and eliminated the second. Therefore, the use of the Thiokol R. D. mount appears desirable in systems with high noise transmission at low frequencies or in systems in which low frequency noise suppression is paramount. The stiffness of Thiokol R. D. increases exponentially with frequency (18). This decreases the effectiveness of Thiokol R. D. in suppressing vibration transmission at higher frequencies. The development of a material to correct this undesirable trend of Thiokol R. D. is recommended.

4.2.2 Foundation

Of the various foundation parameters the mass most affected the vibration transmission. Three different foundation masses were investigated with the result that vibration transmission was lowered as the foundation mass increased. Within the limitations of weight and space, foundations of vibrating machinery should be made as massive as possible.

Viscous foundation damping and damping utilizing the Thiokol R. D. material were studied. Foundation damping was effective at frequencies above 1000 cps for the viscous type and above 800 cps for the Thiokol R. D. type. However, the vibration transmission was reduced more by the Thiokol R. D. type.

Foundation damping is recommended in installations where high frequency noise transmission is a problem.

4.2.3 Hull

The effects of increasing the hull damping first became apparent in the mathematical model used when the damping was increased by an order of twenty. As hull damping was further increased, the vibration transmission was reduced at all frequencies.

These results are at best qualitative. In this model lumped

parameter damping was used, and no attempt was made to study the effects of distributed damping. In practice the latter would be the type applied to the hull. A study to quantitatively evaluate the effects of distributed hull damping would be useful in the extension of an analytic study.

4.2.4 Combined Changes

In a series of cases studied the Portsmouth mount stiffness was decreased, the mount damping increased, and hull damping increased twenty times. This was done with the four systems studied. In general, as the masses in the systems were increased, the vibration transmission was reduced. The results of this series study were compared with those cases in which mount stiffness alone was changed. This comparison showed that the change in mount stiffness overrode the changes in mount damping and hull damping.

A single case of decreased Portsmouth mount stiffness and increased mount damping showed that the vibration transmission was lowered by exactly the same amount as the case in which stiffness alone was decreased. This further supported the conclusion that mount stiffness was one of the most important factors to be considered in any vibration transmission study.

When a mount of Thiokol R. D. was placed in parallel with a Portsmouth mount, the transmission reduction properties were not improved. The addition of the Thiokol R. D. mount actually increased the transmission at frequencies greater than 1000 cps. This case was again studied with the Portsmouth mount stiffness decreased. Results showed that this case was not as good as that in which the single Portsmouth mount with decreased stiffness was examined. Therefore, it is concluded that placing the Thiokol R. D. mounting in parallel with a Portsmouth mounting would not be advantageous in reducing vibration transmission. If parallel mountings are to be used, as recommended by Snowden in Reference (18), care must be

used in the selection of the high damping synthetic rubber material.

In another series of cases the Portsmouth mount was replaced by a Thiokol R. D. mount and Thiokol R. D. damping was applied to the foundation. This was done with the four systems studied. As the masses in the system were increased, the vibration transmission decreased markedly. In any one system the combination of Thiokol R. D. mount and Thiokol R. D. foundation damping was most effective in reducing transmission. It is recommended that this combination be investigated in an experimental installation.

A Portsmouth mount with decreased stiffness and Thiokol R. D. foundation damping comprised the changes examined in another series of cases. Again this was done with the four systems. The vibration transmission was significantly decreased as the masses in the system were increased. Within a particular system this combination was less effective at frequencies of less than 40 cps than the Thiokol R. D. mount and Thiokol R. D. foundation damping combination. However, it was superior at frequencies higher than 40 cps. This combination should also be investigated in an experimental installation.

A Portsmouth mount with decreased stiffness and increased damping and Thiokol R. D. foundation damping comprised the changes in the final series examined. The only transmission change from the previous series was the elimination of the first transmission peak.

4.3 General Considerations

The results of the systematic study largely verified existing concepts in reducing vibration transmission in shipboard machinery installations. This would indicate that the assumptions and the four pole parameter method of analysis were sufficient to establish the vibration trends and characteristics proposed at the beginning of this thesis.

As new ideas for reducing vibration transmission are introduced, it is proposed that they be investigated by means of the technique of this thesis.

The four pole parameter technique enables use of empirical data obtained independently for each mechanical element in the vibration path. Empirical data can be used for elements which defy analytic description. It would appear that the use of empirical data for the element four poles would more closely approximate the vibration characteristics of an actual system.

An interesting extension of this thesis would be an optimization study to determine the best combination of elements in the vibration transmission paths. Since shock specifications also govern the element selection, the optimization must include shock as well as vibration reduction considerations.

The construction of a model of the approximate system, as represented in Figure A-3 of Appendix A, would permit an experimental verification of the four pole parameter technique.

CHAPTER V

CONCLUSIONS

As logically developed in Chapter IV the following statements are listed as the conclusions of this thesis:

1. The four pole parameter technique provides a sufficiently powerful method of analyzing vibration transmission. Furthermore, the four pole parameter technique is easily adapted to a digital computer program.

2. For the purpose of vibration analysis a complex machinery installation can be represented by a relatively simplified system. This representation is sufficient to establish vibration trends and characteristics.

3. The stiffness of the resilient mounting has a dominant effect on the vibration transmission at all frequencies.

4. The only effect of increased damping in the Portsmouth mount was to lower the first vibration transmission peak.

5. A mount fabricated from a high damping synthetic rubber such as Thiokol R. D. is highly effective in reducing vibration transmission at low frequencies. The same mount is not as effective as the Portsmouth mount at higher frequencies.

6. As the mass of the foundation is increased, the vibration transmission is lowered.

7. Foundation damping was effective at frequencies above 1000 cps for the viscous type and 800 cps for the Thiokol R. D. type. The Thiokol R. D. type was more effective.

8. Increasing hull damping decreases vibration transmission at all frequencies.

9. Placing a Thiokol R. D. type mounting in parallel with a

Portsmouth mount as suggested in Reference (18) actually had a detrimental effect on vibration reduction.

10. Replacing a normal mount by a Thiokol R. D. mount and applying Thiokol R. D. damping to the foundation were most effective in reducing vibration transmission.

11. Decreasing the stiffness of a Portsmouth mount and applying Thiokol R. D. damping to the foundation were superior to the system mentioned in the previous conclusion in reducing vibrations at frequencies above 40 cps.

12. A Portsmouth mount with decreased stiffness and increased damping and Thiokol R. D. damping applied to the foundation were the most effective combination in overall transmission reduction.

CHAPTER VI

RECOMMENDATIONS

As stated or inferred in Chapter IV the following are the recommendations of this thesis:

1. The four pole parameter method of analysis should be considered in future design investigations involving vibration transmission.
2. As new ideas for reducing vibration transmission are introduced, they should be investigated by means of the technique of this thesis.
3. Empirical data should be used for element four pole parameters to more closely approximate the vibration characteristics of an actual system.
4. An optimization study to determine the best combination of elements in the vibration transmission paths should be conducted. It should include shock as well as vibration considerations.
5. A model of the approximate system of this thesis should be constructed to permit experimental verification of the four pole parameter technique.
6. Mount stiffness should be as low as possible and yet consistent with shock, alignment, and other factors influencing the design.
7. The material damping of the Portsmouth mount should be increased, or an external damping device placed in parallel with the mount.
8. The Thiokol R. D. mount should be used in systems with high noise transmission at low frequencies or in systems in which low frequency noise suppression is paramount.
9. A high damping mounting material should be developed in which stiffness remains constant or decreases with frequency.

10. Within the limitations of weight and space, foundations of vibrating machinery should be as massive as possible.

11. Foundation damping should be utilized in installations where high frequency noise transmission is a problem.

12. A study to quantitatively evaluate the effects of distributed hull damping should be conducted.

13. Caution should be exercised in the selection of a high damping synthetic rubber mount to be placed in parallel with a standard mount.

14. The effects of replacing a normal mount by a Thiokol R. D. mount and applying Thiokol R. D. damping to the foundation should be investigated in an experimental installation.

15. The effects of decreasing the stiffness of a Portsmouth mount and applying Thiokol R. D. damping to the foundation should be investigated in an experimental installation.

16. The effects of decreasing the stiffness and increasing the damping of a Portsmouth mount and applying Thiokol R. D. damping to the foundation should be investigated in an experimental installation.

APPENDIX A

DETAILS OF PROCEDURE

A-1 The Beam-Mounting System

A-1.1 Experiment

This experiment was conducted to determine the isolation characteristics of a typical Admiralty mounting. The experimental system consisted of a fifty pound beam driven by a harmonic vibration generator. The beam was supported by the mounting which rested on a "rigid" foundation. The components and instrumentation are illustrated in Figure A-1.

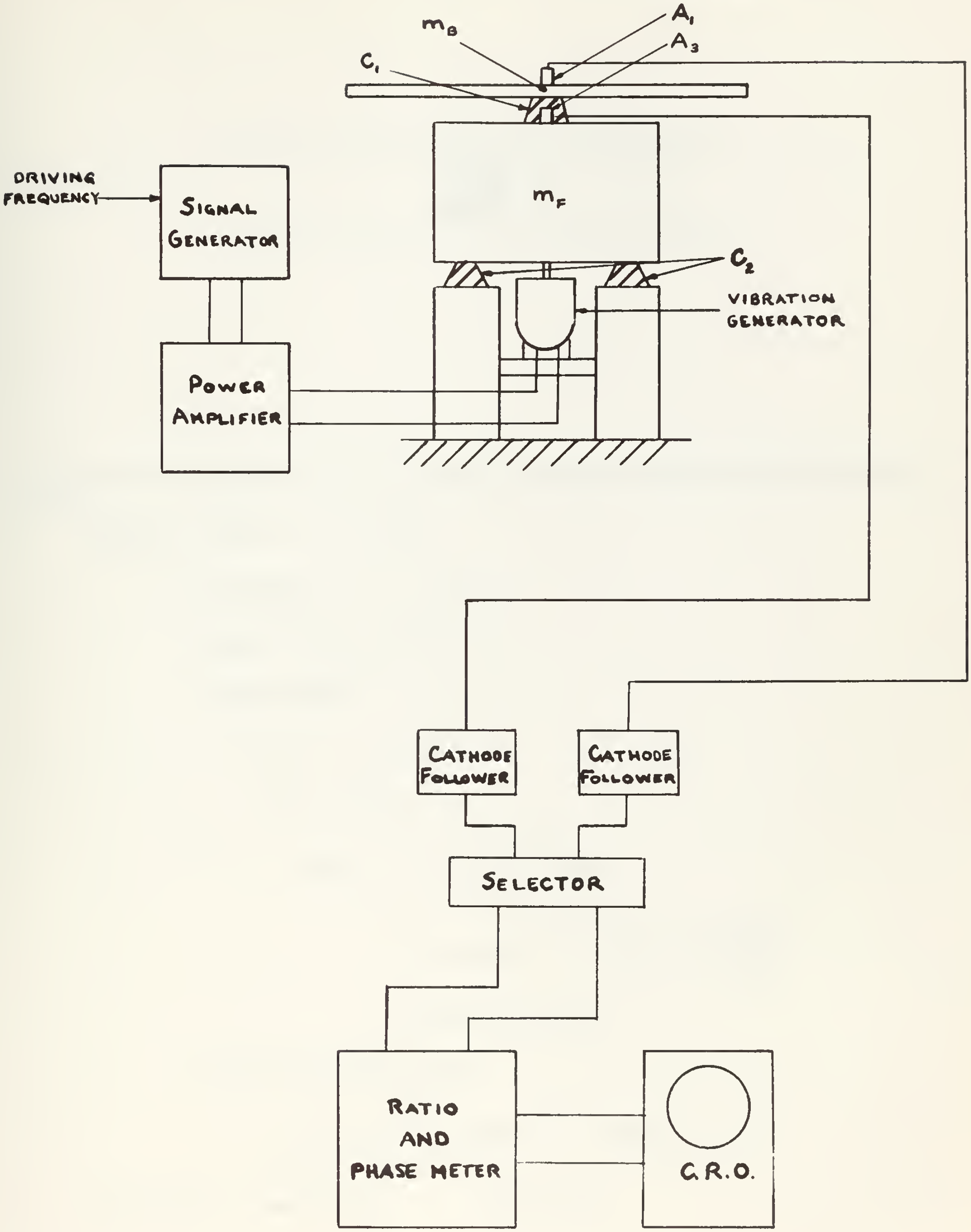
The foundation system was not, in fact, rigid. However, its natural frequency was much lower than the rigid body mode frequency of the beam-mounting system. This insured no resonance interaction, and the foundation mass, m_F in Figure A-1, responded in a rigid manner. The vibration generator imparted a velocity to the mass m_F. The velocity V_3 into the mounting of compliance C_1 and the velocity V_1 at the free side of the beam were picked up by the barium titanate accelerometers, A_3 and A_1 respectively. The isolation in this case is the ratio of V_2 to V_1. Both this and the driving frequency were recorded, and the isolation was converted to decibels. This was repeated over the frequency range of 10 to 3000 cps.

To obtain a more realistic model in which the beam was driven from above, as a machine normally would be, use of the transmissibility theorem was necessary. This stated that the transmission (or isolation) in one direction for linear elastic systems is equivalent to the force transmission in the opposite direction:

$$I = \frac{V_2}{V_1} = \frac{F_1}{F_2}$$

This new system could be represented:

FIGURE A-1
PRELIMINARY EXPERIMENT

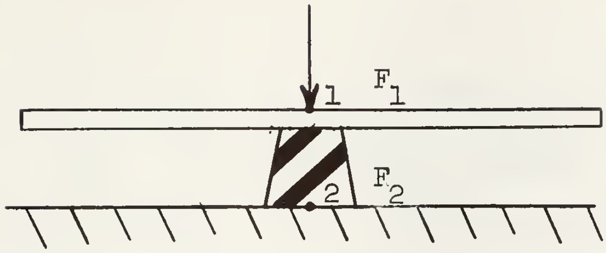


A_1, A_2 - BARIUM TITANATE ACCELEROMETERS

C_1, C_2 - MOUNTING COMPLIANCES

m_B - MASS OF BEAM

m_F - MASS OF FOUNDATION



The physical constants of the system of the experiment were as follows:

Beam: Length: $L = 60$ in.

Thickness: $t = .75$ in.

Weight: $W_B = 50$ lb.

Frequencies:

$$\omega_n = \lambda_n \sqrt{\frac{EI}{m_B}}$$

where: $E = 30 \times 10^6$ psi

I = moment of inertia $= \frac{W}{12}\frac{t^2}{\rho L}$

ρ = mass density

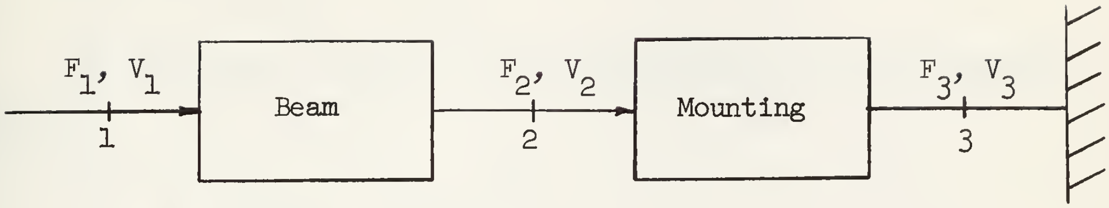
Describing Functions:

$\phi_n\left(\frac{L}{2}\right)$ and λ_n given in Reference (16)

Mounting: Compliance: $C_1 = \frac{1}{1060}\frac{\text{in.}}{\text{lb.}}$

A-1.2 Theory

The beam-mounting system was represented schematically:



$[S_1]$ was the matrix of four pole parameters for the beam and $[S_2]$ for the mounting.

Then:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} S_1 \end{bmatrix} \begin{bmatrix} S_2 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (1)$$

or by individual steps:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} S_1 \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (1a)$$

$$\begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} S_2 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \quad (1b)$$

The matrices in terms of their four poles were expressed:

$$\begin{bmatrix} S_1 \end{bmatrix} = \begin{bmatrix} {}^1\epsilon_{11} & {}^1\epsilon_{12} \\ {}^1\epsilon_{21} & {}^1\epsilon_{22} \end{bmatrix} \quad (3)$$

and

$$\begin{bmatrix} S_2 \end{bmatrix} = \begin{bmatrix} {}^{2}\epsilon_{11} & {}^{2}\epsilon_{12} \\ {}^{2}\epsilon_{21} & {}^{2}\epsilon_{22} \end{bmatrix} \tag{4}$$

As developed in Section 2.2.2, the general four pole parameter matrix is:

$$\begin{bmatrix} S_n \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{T_F}\right)^b & \left(\frac{1}{M_{12}}\right)^f \\ (M_{21})^b & \left(\frac{1}{T_V}\right)^f \end{bmatrix}$$

In this proof of the experiment the mounting was treated as a simple spring. From Appendix A-2, the four poles for a simple spring are:

$$\begin{bmatrix} S_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C_1 & 1 \end{bmatrix} \tag{5}$$

The compliance of the spring is C_1, while the mobility is $j\omega C_1$.

The beam was analyzed following the techniques of Ruzicka and Cavanaugh (4). With negligible compression assumed in the transverse direction and from the transmissibility theory, the four poles of the beam were evaluated:

$$\left(\frac{1}{T_F}\right)^b = \left(\frac{1}{T_V}\right)^f = 1$$

$$(M_{21})^b = 0$$

$$\left(\frac{1}{M_{12}}\right)^f = \frac{1}{M(\frac{L}{2})}$$

where $M(\frac{L}{2})$ was the mobility at the center of the beam. This was taken from Reference (4):

$$M\left(\frac{L}{2}\right) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2\left(\frac{L}{2}\right)}{\omega_n^2 - \omega^2} - \frac{j}{m_B \omega} \qquad (6)$$

The matrix $[S_1]$ for the beam was then written:

$$\begin{bmatrix} S_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{M\left(\frac{L}{2}\right)} \\ 0 & 1 \end{bmatrix} \qquad (7)$$

Equations (5) and (7) were substituted in equation (1):

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{M\left(\frac{L}{2}\right)} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C_1 & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \qquad (8)$$

To check the experimental results a plot of isolation I, which is defined as $\frac{F_1}{F_3}$, versus frequency was desired. Equation (8) was rewritten:

$$F_1 = \left(1 + \frac{j\omega C_1}{M\left(\frac{L}{2}\right)}\right) F_3 + \frac{V_3}{M\left(\frac{L}{2}\right)}$$

$$V_1 = (j\omega C_1)\, F_3 + V_3$$

Station 3 in the experiment was a rigid support; therefore, $V_3 = 0$.

The isolation was obtained:

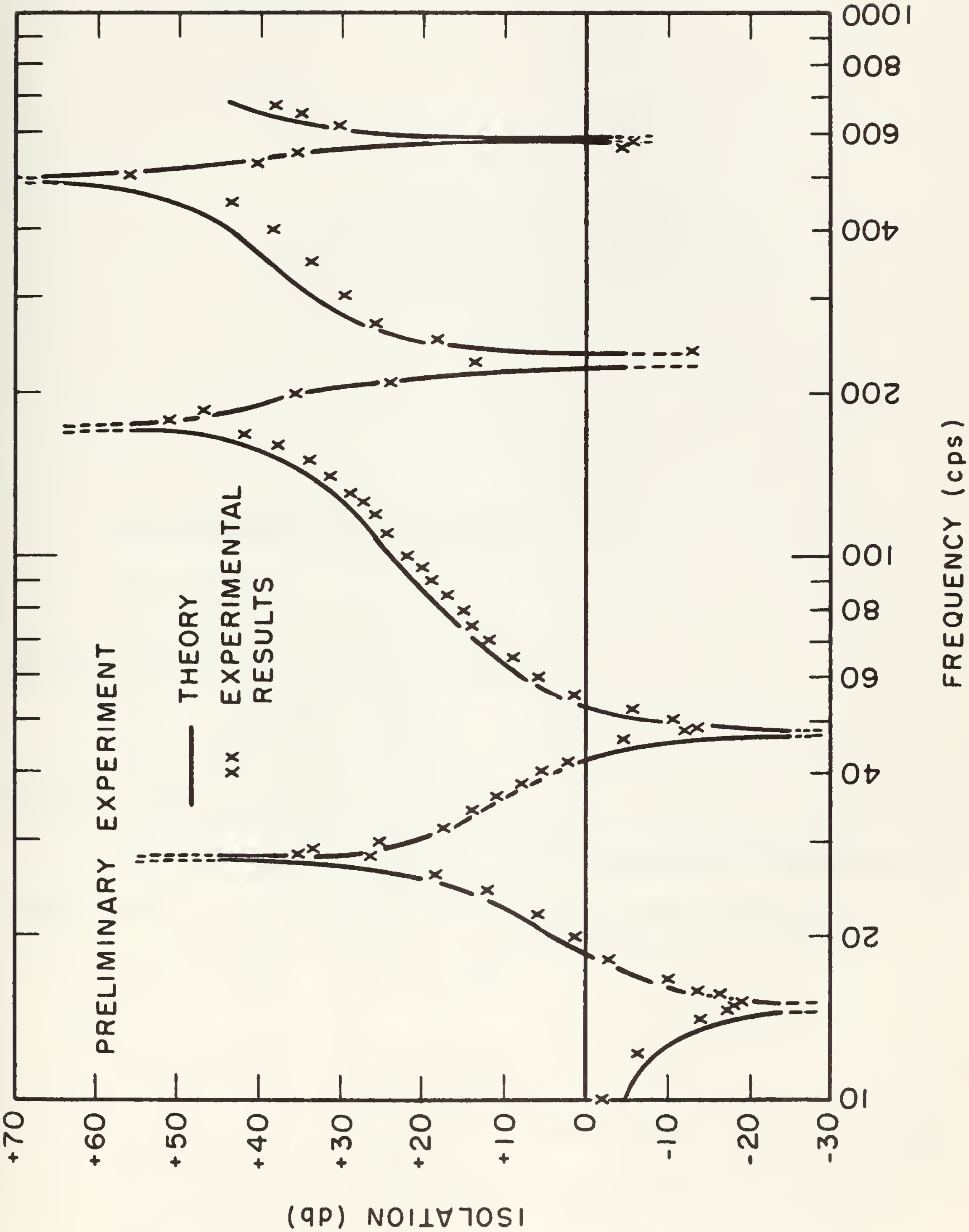
$$I = \frac{F_1}{F_3} = 1 + \frac{j\omega C_1}{M\left(\frac{L}{2}\right)}$$

The mobility of the beam is expressed in Equation (6).

A-1.3 Comparison of Experiment and Theory

The curves of isolation in Figure A-2 show that the four pole parameter technique provided excellent theoretical verification of the experimentally measured values of isolation. These results indicated the value of the four pole analysis and suggested the use of it in the study

FIGURE A-2
PRELIMINARY EXPERIMENT



of more complex systems.

A-2 Development of Four Poles

A-2.1 Mass Four Poles

The performance equations are

$$V_1 = V_2 \tag{1}$$

because the mass is a rigid body and

$$F_1 - F_2 = m \frac{dV_1}{dt} = m \frac{dV_2}{dt} \tag{2}$$

from the application of Newton's Law.

Since the applied force is usually considered as harmonic, the other force and the velocities will also be harmonic functions.

$$F_1 = F_{10}\ e^{j\omega t} \qquad F_2 = F_{20}\ e^{j\omega t}$$

$$V_1 = V_{10}\ e^{j\omega t} \qquad V_2 = V_{20}\ e^{j\omega t}$$

F_{10}, F_{20}, V_{10}, and V_{20} are time independent but may be complex.

Substituting the expression for V_1, Equation (2) becomes:

$$F_1 = F_2 + (j\omega m)\ V_2$$

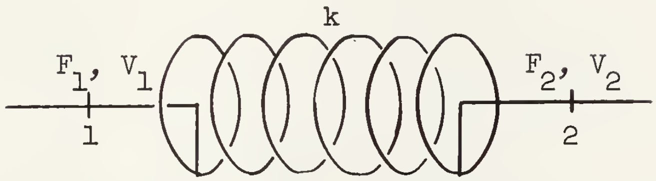
while (1) remains

$$V_1 = V_2$$

By inspection the four pole parameters are:

$\epsilon_{11} = 1$, $\epsilon_{12} = j\omega m$, $\epsilon_{21} = 0$, and $\epsilon_{22} = 1$.

A-2.2 Spring Four Poles



The performance equations are

$$F_1 = F_2 \tag{1}$$

because the spring possesses no inertia and

$$F_1 = k\,[x_1 - x_2] \quad \text{where } x \text{ is displacement.} \tag{2}$$

The velocity is the time derivative of the displacement.

From the harmonic velocity expressions of the previous section:

$$V_1 = V_{10}\,e^{j\omega t} \qquad V_2 = V_{20}\,e^{j\omega t}$$

and

$$x_1 = \int V_{10}\,e^{j\omega t}\,dt \qquad x_2 = \int V_{20}\,e^{j\omega t}\,dt$$

or

$$x_1 = \frac{V_1}{j\omega} \qquad x_2 = \frac{V_2}{j\omega}$$

Equations (1) and (2) are rewritten:

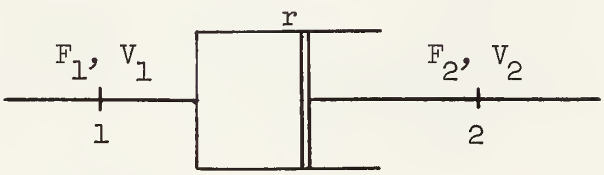
$$F_1 = F_2$$

$$V_1 = \frac{j\omega}{k} F_2 + V_2$$

By inspection the four pole parameters are:

$\epsilon_{11} = 1$, $\epsilon_{12} = 0$, $\epsilon_{21} = \frac{j\omega}{k}$, and $\epsilon_{22} = 1$.

A-2.3 Damper Four Poles



The performance equations are

$$F_1 = F_2 \tag{1}$$

because the damper possesses no inertia and

$$F_1 = r(V_1 - V_2) \tag{2}$$

from the definition of damping.

These are rewritten

$$F_1 = F_2$$

$$V_1 = \frac{1}{r} F_2 + V_2.$$

By inspection the four poles are

$\epsilon_{11} = 1$, $\epsilon_{12} = 0$, $\epsilon_{21} = \frac{1}{r}$, and $\epsilon_{22} = 1$.

A-3 <u>Parallel Matrix Combination</u>

If n systems are connected in parallel then a single four pole

matrix can be written connecting the common input and output points. If this matrix is written $[S_p]$, then

$$\begin{bmatrix} S_p \end{bmatrix} = \begin{bmatrix} \frac{A}{B} & \frac{AC}{B} - B \\ \frac{1}{B} & \frac{C}{B} \end{bmatrix} \tag{1}$$

Let the r^{th} matrix, $[S_r]$, be given by:

$$\begin{bmatrix} S_r \end{bmatrix} = \begin{bmatrix} {}^{r}\epsilon_{11} & {}^{r}\epsilon_{12} \\ {}^{r}\epsilon_{21} & {}^{r}\epsilon_{22} \end{bmatrix}, \tag{2}$$

where r varies from 1 to n. Then, in (1) above

$$A = \sum_{r=1}^{n} \left[\frac{{}^{r}\epsilon_{11}}{{}^{r}\epsilon_{21}} \right] \tag{3}$$

$$B = \sum_{r=1}^{n} \left[\frac{1}{{}^{r}\epsilon_{21}} \right] \tag{4}$$

$$C = \sum_{r=1}^{n} \left[\frac{{}^{r}\epsilon_{22}}{{}^{r}\epsilon_{21}} \right] \tag{5}$$

A more complete discussion is given in reference (6).

A-4 Development of Four Poles of General System

A-4.1 The Machine

As stated in Section 2.4.1 the machine was represented by a uniform elastic beam driven by a harmonic force generator. This system was selected because it was multi-resonant and possessed distributed parameters.

The four poles of this system are those briefly described in Appendix A-1.2 for the beam in the preliminary experiment. These four poles were $\epsilon_{11} = 1$, $\epsilon_{12} = \frac{1}{M(\frac{L}{2})}$, $\epsilon_{21} = 0$, and $\epsilon_{22} = 1$. From the method

of Reference (4), the beam mobility $M(\frac{L}{2})$ will be derived.

The basic equation of free vibration is:

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \tag{1}$$

where: I = moment of inertia of beam's cross section

E = modulus of elasticity

ρ = mass density

A = beam cross-section area

y = transverse displacement

x = distance along length of beam

L = length of beam

t = time

Assuming a harmonic solution:

$$y = f(x)e^{j\omega t} \tag{2}$$

If this is substituted, equation (1) becomes:

$$\frac{\partial^4 f}{\partial x^4} - \beta^4 f = 0 \tag{3}$$

where

$$\beta^4 = \frac{\rho A \omega^2}{EI}$$

and ω = frequency of vibration.

The general solution of (3) is:

$$f(x) = C_1 (\sinh \beta x + \sin \beta x) + C_2 (\cosh \beta x + \cos \beta x) + C_3 (\sinh \beta x - \sin \beta x) + C_4 (\cosh \beta x - \cos \beta x) \tag{4}$$

The boundary conditions for a free-free beam are:

$$\frac{d^2 f(0)}{dx^2} = 0 \tag{5}$$

$$\frac{d^2 f(L)}{dx^2} = 0 \tag{6}$$

$$\frac{d^3 f(0)}{dx^3} = 0 \tag{7}$$

$$\frac{d^3 f(L)}{dx^3} = 0 \tag{8}$$

Equations (5) and (6) require no shear forces at the ends of the beam; equations (7) and (8) state that there are no end moments.

When the boundary conditions are applied to equation (4), the characteristic frequency equation is obtained.

$$\cos \beta_n L \cosh \beta_n L = 1 \tag{9}$$

This is a transcendental equation. Corresponding to each eigenvalue β_n will be a value of $f_n(x)$. The term $f_n(x)$ represents the mode shape in which the beam vibrates at the frequency defined by β_n. The relationship between $f_n(x)$ and β_n is:

$$f_n(x) = C_2[\cosh \beta_n x + \cos \beta_n - \alpha_n(\sinh \beta_n x + \sin \beta_n x)] \tag{10}$$

where

$$\alpha_n = \frac{\sinh \beta_n L + \sin \beta_n L}{\cosh \beta_n L - \cos \beta_n L} \tag{11}$$

C_2 = arbitrary constant

Define

$$f_n(x) = C_2 \phi_n(x).$$

Then in equation (10):

$$\phi_n(x) = \cosh \beta_n x + \cos \beta_n x - \alpha_n (\sinh \beta_n x + \sin \beta_n x)$$

The term $\phi_n(x)$ will satisfy the equation:

$$\frac{d^4\phi_n}{dx^4} = \beta_n^4 \phi_n$$

It is a normal function which forms an orthogonal set in the interval $0 \leq x \leq L$. The condition of orthogonality:

$$\int_0^L \phi_n \phi_m \, dx = 0 \qquad m \neq n$$

However, when n = m:

$$\int_0^L \phi_n^2 (x) \, dx = \frac{L}{4} [\phi_n (L)]^2$$

D. Young and R. Felgar have evaluated these ϕ functions for beams with a variety of boundary conditions (16).

With the values of ϕ_n and the corresponding values of ω_n, the forced vibration motion is considered.

If a harmonic force $F_0 e^{j\omega t}$ is applied to the beam, the displacement according to reference (1) is:

$$y(x) = \frac{F_0 e^{j\omega t}}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2 (x)}{\omega_n^2 - \omega^2} + \text{rigid body motion} \qquad (12)$$

The rigid body motion is $f_0 (x) e^{j\omega t}$. The velocity is the derivative of the displacement:

$$\dot{y}(x) = \frac{j\omega F_0 e^{j\omega t}}{m_B} \sum_{n=1}^{\infty} \frac{{}_n^2 (x)}{\omega_n^2 - \omega^2} + j\omega f_0 (x) e^{j\omega t} \qquad (13)$$

From the definition of mobility at the driving point p:

$$M = \frac{V(p)}{F(p)} = \frac{\dot{y}(p)}{F_0 e^{j\omega t}}$$

$$M(p) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2(p)}{\omega_n^2 - \omega^2} + \frac{j\omega f_0}{F_0} \tag{14}$$

As a rigid body the beam acts like a mass and possesses the mobility of this lumped parameter. From Appendix A-2.1:

$$\text{Mass Mobility} = \frac{-j}{m_B \omega}$$

Therefore, in equation (14):

$$\frac{j\omega f_0}{F_0} = \frac{-j}{m_B \omega}$$

The final expression for beam mobility is:

$$M(p) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2(p)}{\omega_n^2 - \omega^2} - \frac{j}{m_B \omega} \tag{15}$$

Since the beam in the machine representation was considered to be driven at its center, the four pole ϵ_{12} required the inverse mobility at the center of the beam. This was expressed from equation (15) as:

$$M(\tfrac{L}{2}) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2(\frac{L}{2})}{\omega_n^2 - \omega^2} - \frac{j}{m_B \omega} .$$

The machine could now be represented in terms of its four poles.

A-4.2 The Mounting

According to Section 2.4.2, two different mountings were studied. The first was the standard Portsmouth mounting which was analyzed following the method of Wright and Hagg (17).

The empirical results of tests on actual mountings were used by Wright and Hagg. The mount characteristics were approximated by use of the

electrical impedance analogy, one used extensively in vibration analysis (8). The mounting was represented by a shunt impedance consisting of real and imaginary parts. In the language of the analogy, impedance is defined as a force to velocity ratio.

$$\bar{Z} = \frac{F}{V} \tag{1}$$

In this case input impedance was considered for the approximation. From the analogy mechanical damping corresponds to electrical resistance or the real part of the impedance. Mass and spring-like properties correspond to electrical reactance or the imaginary part of the impedance. Thus the approximation included all the important lumped parameters.

These parameters, damping constant R and spring stiffness K, were established to be:

$$R = \frac{K}{2\pi f Q_0} \tag{2}$$

$$K = \frac{4\pi^2 \, W f_0^2 \left(1 + \frac{\pi f}{f_0}\right)}{g\left(1 + \frac{\pi f_0}{f_t}\right)} \tag{3}$$

where:

f_0 = rated natural frequency of the mount at its rated load, cps, empirically determined.

Q_0 = maximum transmissibility at f_0, dimensionless, empirically determined.

f_t = frequency of the first high frequency transmission peak, cps, empirically determined.

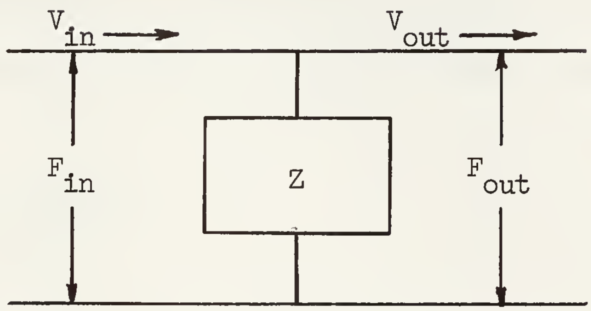
$\frac{W}{g}$ = rated mass load on the mount, lbm, empirically determined.

f = driving or exciting frequency, cps.

The shunt impedance Z is therefore given by:

$$Z = R + \frac{K}{j\omega} \tag{4}$$

The impedance analogy indicates the following equations hold for a shunt impedance:



$$F_{in} = F_{out} \tag{5}$$

$$Z = \frac{F_{out}}{V_{in}}, \text{ open circuit} \tag{6}$$

$$V_{in} = V_{out}, \text{ short circuit} \tag{7}$$

Equations (5) to (7) are translated into four pole parameter notation as in Section 2.2:

$$F_1 = F_2 \tag{8}$$

$$\frac{1}{(M_{21})^b} = Z = \frac{F_2}{V_1} \tag{9}$$

$$V_1 = V_2 \quad \text{station 2 free} \tag{10}$$

Therefore, the four pole parameters for this standard Portsmouth mounting are

$$\epsilon_{11} = 1, \ \epsilon_{12} = 0, \ \epsilon_{21} = \frac{1}{Z}, \text{ and } \epsilon_{22} = 1.$$

Combining equations (2), (3), and (4) the following expression for ϵ_{21} is obtained.

$$\epsilon_{21} = \frac{\omega Q_0}{K} \left[\frac{1 + jQ_0}{Q_0^2 + 1}\right] \qquad (11)$$

where ω is the forcing frequency, rad/sec.

The second mounting considered was treated similarly except that different expressions were used for R and K. These were derived by A. O. Sykes (14):

$$R = \frac{2rG}{\omega(1 - r^2)} \left(\frac{S}{L}\right) \qquad (12)$$

$$K = G\left(\frac{S}{L}\right) \qquad (13)$$

where:

S = mounting cross-sectional area

L = mounting length

G = mounting elastic modulus

r = mounting damping factor

ω = forcing frequency

The elastic modulus and damping factor were given as empirical results for a synthetic high damping rubber called Thiokol R. D. by J. C. Snowden (18).

Approximate formulae were determined for G and r from the empirical data:

$$G = 72.6\,\omega^{0.4} \qquad (14)$$

$$r = 0.1\,\omega^{0.325} \qquad (15)$$

Thus R and K could be represented as functions of frequency and the new four pole parameter ϵ_{21} evaluated.

$$\epsilon_{21} = \frac{\omega}{G\left(\frac{S}{L}\right)(A^2 + 1)}\,[A + j]$$

where

$$A = \frac{2r}{1 - r^2}$$

A-4.3 The Foundation

As stated in Section 2.4.3, the foundation was considered a beam resting on two columnar structures. The two columns presented two possible, identical paths for vibration transmission. Since the paths were identical, only one was studied explicitly.

The four poles of the foundation structure were a series combination of the individual four poles of a beam and column.

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{M(L)} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ M_C & 1 \end{bmatrix}$$

The combined element four poles are

$$\epsilon_{11} = 1 + \frac{M_C}{M(L)}, \ \epsilon_{12} = \frac{1}{M(L)}, \ \epsilon_{21} = M_C, \text{ and } \epsilon_{22} = 1.$$

The term M(L) is the transfer mobility of the beam relating the driving point with the junction of the beam and column structure. Initially, this beam was considered as end-supported for the purposes of deriving its mobility. Correspondence with the Admiralty Engineering Laboratory, where an experiment driving an end-supported beam was being conducted, indicated that for vibration analysis the end-supported beam behaved more like a free-free beam. For this reason the foundation beam was treated much as the beam in Appendix A-4.1. The expression for beam mobility is:

$$M(L) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n(\frac{L}{2})\phi_n(L)}{\omega_n^2 - \omega^2} - \frac{j}{m_B \omega}$$

where the ϕ's are the normal functions at the driven point $(\frac{L}{2})$ and the beam column junction (L). Again these functions were taken from Reference (16).

From Reference (1) the expression for the lateral deflection of a column being driven by a harmonic force $F_0 e^{j\omega t}$ is:

$$y(x) = f_0 e^{j\omega t} + \sum_{n=1}^{\infty} \frac{F_0}{\rho LA} \frac{2(-1)^{\frac{n-1}{2}} e^{j\omega t}}{(\omega_n^2 - \omega^2)}$$

where:

ρ = mass density

L = column length

A = column cross section area

The column mobility can be derived using the same methods used for the beam in Appendix A-4.1. The velocity is:

$$\dot{y}(x) = j\omega f_0 e^{j\omega t} + \frac{j\omega F_0 e^{j\omega t}}{\rho LA} \sum_{n=1}^{\infty} \frac{2(-1)^{\frac{n-1}{2}}}{(\omega_n^2 - \omega^2)}$$

Now the definitions of mobility and the rigid mass mobility are applied. From this the mobility of the column is obtained:

$$M_C = \frac{-j}{m_c \omega} + \frac{j\omega}{\rho LA} \sum_{n=1}^{\infty} \frac{2(-1)^{\frac{n-1}{2}}}{(\omega_n^2 - \omega^2)}$$

A-4.4 The Hull

As stated in Section 2.4.4, the work of D. V. Wright was consulted again for a workable analytic representation of the ship's hull (7).

Wright considered the hull a series impedance, which he represented as a result of analyzing empirical data obtained from typical hull impedance measurements (11, 20). As in the case of the mounting the impedance consisted of a real and imaginary part, the resistive and reactive properties. These were represented as complicated quantities dependent on nine separate lumped parameters.

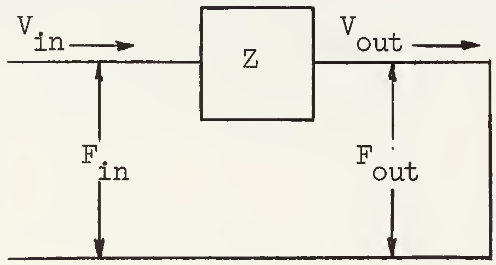
$$R = R_s + R_1 f + \frac{R_2}{f} + \frac{R_p}{1 + R_p^2 \left(\frac{193}{\pi W_p f} - \frac{2\pi f}{K_p}\right)^2} \qquad (1)$$

$$X = X_s + \frac{\pi W_s f}{193} - \frac{K_s}{2\pi f} + \frac{R_p^2 \left(\frac{193}{\pi W_p f} - \frac{2\pi f}{K_p}\right)}{1 + R_p^2 \left(\frac{193}{\pi W_p f} - \frac{2\pi f}{K_p}\right)^2} \qquad (2)$$

$$Z = R + jX \qquad (3)$$

where R_s, R_1, R_2, R_p, K_s, K_p, W_s, W_p are the nine lumped parameters and f is the driving frequency.

The nine parameters were considered circuit elements in the impedance analogy, making up the series impedance Z.



The impedance analogy gives:

$$Z = \frac{F_{in}}{V_{in}} \qquad (4)$$

$$V_{in} = V_{out} \qquad (5)$$

$$F_{in} = F_{out}, \text{ open circuit} \qquad (6)$$

Equations (4) through (6) are translated into four pole parameter notation by the method of Section 2.2:

$$V_1 = V_2 \qquad (7)$$

$$F_1 = F_2 \quad \text{station 2 blocked} \qquad (8)$$

$$\frac{1}{(M_{12})^f} = Z = \frac{F_1}{V_2} \qquad (9)$$

The four pole parameters therefore become:

$$\epsilon_{11} = 1 \qquad (10)$$

$$\epsilon_{12} = Z \qquad (11)$$

$$\epsilon_{21} = 0 \qquad (12)$$

$$\epsilon_{22} = 1 \qquad (13)$$

A-4.5 The Water

As mentioned in Section 2.4.5 the water, considered only as a resistance to the propagation of acoustic waves, has a specific resistance R* dependent on the water density and propagation velocity. This can be written, (2):

$$R^* = \frac{R}{A} = \rho c \qquad (1)$$

where

R^* = Specific resistance

R = Resistance

A = Effective radiating area

ρ = density

c = velocity of sound in water

Treating the water as a dashpot-type resistance (velocity proportional), the four pole parameters were obtained in Appendix A-2.3.

$$\epsilon_{11} = 1 \qquad (2)$$

$$\epsilon_{12} = 0 \qquad (3)$$

$$\epsilon_{21} = \frac{1}{R} \qquad (4)$$

$$\epsilon_{22} = 1 \qquad (5)$$

R was evaluated by two methods. In one an effective area A was assumed constant with frequency.

$$R = \rho c A \tag{6}$$

The hull was considered also as a vibrating diaphragm. The radius r of the effective vibrating area is given by:

$$r = \frac{c}{\omega} \tag{7}$$

where ω is the radiating frequency.

Therefore:

$$R = \frac{\pi \rho c^3}{\omega^2} \tag{8}$$

Substituting (6) or (8) into (4) gave the desired four pole parameters.

A-5 System Properties and Systematic Changes

A-5.1 General Description

Four variations of the general system of Figure A-3 were analyzed in this study. These variations were made in the relative sizes of the machine and foundation structures to minimize any scale effect in the trends of vibration transmission. Within each of these systems, element properties were varied. In addition, isolation elements were inserted at various locations in the systems. From these variations and insertions, trends to provide a guide for preferred design practice in the reduction of vibration transmission were established.

A-5.2 System 1

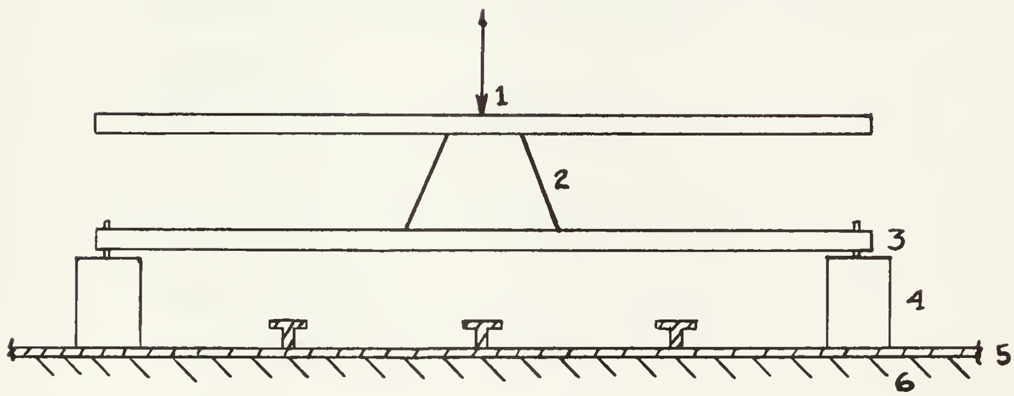
With reference to Figure A-3 the properties of System 1 were:

1. Machine

   Weight of beam, W_B = 50.0 lbf

   Mass of beam, m_B = .1295 lbm

FIGURE A-3
GENERAL SYSTEM



KEY:
1: MACHINE
2: MOUNTING
3: FOUNDATION
4: COLUMN
5: HULL
6: WATER

Length of beam, L = 60.0 in

Thickness of beam, t = .75 in

2. Mounting

Rated natural frequency of the mount at rated load,

$$f_0 = \frac{\omega_0}{2\pi} = 16.0 \text{ cps}$$

Maximum transmission at f_0, $Q_0 = 7.0$

Frequency of the first high frequency transmission peak,

$$f_t = \frac{\omega_t}{2\pi} = 1000 \text{ cps}$$

Rated load of mount, W = 50.0 lbf

3. Foundation

Weight of beam, W_B = 30.0 lbf

Mass of beam, m_B = .078 lbm

Length of beam, L = 36.0 in

Thickness of beam, t = .75 in

4. Column

Circular Cross-section

Length of column, L = 6.0 in

Mass of column, m_c = .039 lbm

Weight of column, W_c = 15.0 lbf

5. Hull

The nine parameters are:

R_s = 50 lbf-sec/in

R_1 = .01 lbf-sec/in

R_p = 5000 lbf-sec/in

X_s = 500 lbf-sec/in

W_s = 10.0 lbf

K_s = 10^6 lbf/in

W_p = 15 lbf

K_p = 2×10^6 lbf/in

6. Water

R = 37,600 lbf-sec/in (constant with frequency)

A-5.3 System 2

With reference to Figure A-3 the properties of System 2 were:

Identical to System 1 except:

Foundation

W_B = 100 lbf

m_B = .259 lbm

L = 60.0 in

t = 1.5 in

A-5.4 System 3

With reference to Figure A-3 the properties of System 3 were:

Identical to System 1 except:

Foundation

W_B = 200 lbf

m_B = .518 lbm

L = 60.0 in

t = 3.00 in

A-5.5 System 4

With reference to Figure A-3 the properties of System 4 were:

Identical to System 1 except:

Machine

$W_B = 500$ lbf

$m_B = 1.295$ lbm

$L = 60.0$ in

$t = 3.75$ in

Foundation

$W_B = 300$ lbf

$m_B = .78$ lbm

$L = 60.0$ in

$t = 2.25$ in

A-5.6 Element Insertions

A-5.6.1 Thiokol R. D. Mounting

In the systematic study the Thiokol R. D. mounting was either substituted for the Portsmouth mounting or placed in parallel with it. The properties of the Thiokol R. D. mounting were:

Mounting cross sectional area, $S = 9.43 \text{ in}^2$

Mounting length, $L = 4.0$ in

A-5.6.2 Foundation Damping

Viscous foundation damping which was constant with frequency was first considered. The damper was inserted between the foundation and the column. The property of the damper was:

System 1:

Damping constant, $R = 390 \dfrac{\text{lbf-sec}}{\text{in}}$

System 2:

$R = 390 \dfrac{\text{lbf-sec}}{\text{in}}$

System 3:

$$R = 1560 \ \frac{\text{lbf-sec}}{\text{in}}$$

System 4:

$$R = 1760 \ \frac{\text{lbf-sec}}{\text{in}}$$

Damping at the same position with Thiokol R. D. as the damping material was next considered. The dimensions of the Thiokol R. D. were:

Cross sectional area, $S = 20.0 \text{ in}^2$

Thickness, $L = 1.0$ in

The thickness of the damping material was such that its compliance property could be neglected.

A-5.7 Systematic Changes

The systematic changes in the structural foundation path are best followed by an examination of the individual cases.

Case 1. System 1 used. This was the basic case for System 1 with which the cases with property changes could be compared.

Case 2. System 1 used. Mount damping was increased by decreasing Q_0 to 2.5.

Case 3. System 1 used. Mount stiffness was increased by setting $\omega_0 = 201.062$ and $\omega_t = 12{,}566.384$.

Case 4. System 1 used. Mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 5. System 1 used. The Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 6. System 1 used. The hull damping was increased by setting $R_s = 100$, $R_1 = .02$, and $R_2 = 2000$.

Case 7. System 1 used. The mount damping was increased by setting

$Q_0 = 2.5$, and the mount stiffness was decreased by settin $\omega_0 = 50.266$ and $\omega_t = 3,141.596$. This case was thus a combination of cases 2 and 4.

Case 8. System 1 used. The Thiokol R. D. mounting was used, and the hull damping was increased by setting $R_s = 100$, $R_1 = .02$, and $R_2 = 1000$. This case was thus a combination of cases 5 and 6.

Case 9. System 1 used. Mount damping was increased, mount stiffness decreased, and hull damping increased. This was accomplished by setting $Q_0 = 2.5$, $\omega_0 = 50.266$, $\omega_t = 3.141.596$, $R_s = 1000$, $R_1 = 1.0$, and $R_2 = 20,000$.

Case 10. System 1 used. Viscous foundation damping was inserted in the system with R = 390.

Case 11. System 1 used. The Thiokol R. D. and the Portsmouth mounting were placed in parallel.

Case 12. System 1 used. The dissipative resistance of the water was varied with frequency. See equation (8) in Appendix A-4.5.

Case 13. System 1 used. The mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3,141.596$, and viscous foundation damping was inserted in the system with R = 390. This case was thus a combination of cases 4 and 10.

Case 14. System 1 used. Foundation damping with Thiokol R. D. was inserted in the system.

Case 15. System 1 used. The Thiokol R. D. and the Portsmouth mounting were placed in parallel. The stiffness of the Portsmouth mounting was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3,141.596$. This case was thus a combination of cases 4 and 11.

Case 16. System 1 used. Hull damping was increased by setting $R_s = 100,000$, $R_1 = 100$, and $R_2 = 2 \times 10^6$.

Case 17. System 1 used. Foundation damping with Thiokol R. D. was inserted, and the Thiokol R. D. mounting replaced the Portsmouth mounting. This case was thus a combination of cases 5 and 14.

Case 18. System 1 used. Foundation damping with Thiokol R. D. was inserted, and mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$. This case was thus a combination of cases 4 and 14.

Case 19. System 1 used. Foundation damping with Thiokol R. D. was inserted, mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$, and mount damping was increased by setting $Q_0 = 2.5$. This case was thus a combination of cases 19 and 2.

Case 20. System 2 used. This was the basic case for System 2 with which the cases with property changes could be compared.

Case 21. System 2 used. Mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 22. System 2 used. The Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 23. System 2 used. Mount damping was increased, mount stiffness decreased, and hull damping increased. This was accomplished by setting $Q_0 = 2.5$, $\omega_0 = 50.266$, $\omega_t = 3{,}141.596$, $R_s = 1000$, $R_1 = 1.0$, and $R_2 = 20{,}000$.

Case 24. System 2 used. Viscous foundation damping was inserted in the system with $R = 390$.

Case 25. System 2 used. Foundation damping with Thiokol R. D. was inserted, and the Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 26. System 2 used. Foundation damping with Thiokol R. D.

was inserted, and mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 30. System 3 used. This was the basic case for System 3 with which the cases with property changes could be compared.

Case 31. System 3 used. Mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 32. System 3 used. The Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 33. System 3 used. Mount damping was increased, mount stiffness decreased, and hull damping increased. This was accomplished by setting $Q_0 = 2.5$, $\omega_0 = 50.266$, $\omega_t = 3{,}141.596$, $R_s = 1000$, $R_1 = 1.0$, and $R_2 = 20{,}000$.

Case 34. System 3 used. Viscous foundation damping was inserted in the system with $R = 1560$.

Case 35. System 3 used. Foundation damping with Thiokol R. D. was inserted, and the Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 36. System 3 used. Foundation damping with Thiokol R. D. was inserted, and mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 50. System 4 used. This was the basic case for System 4 with which the cases with property changes could be compared.

Case 51. System 4 used. Mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3{,}141.596$.

Case 52. System 4 used. The Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 53. System 4 used. Mount damping was increased, mount stiffness decreased, and hull damping increased. This was accomplished by

setting $Q_0 = 2.5$, $\omega_0 = 50.266$, $\omega_t = 3,141.596$, $R_s = 1000$, $R_1 = 1.0$, and $R_2 = 20,000$.

Case 54. System 4 used. Viscous foundation damping was inserted in the system with $R = 1760$.

Case 55. System 4 used. Foundation damping with Thiokol R. D. was inserted, and the Thiokol R. D. mounting replaced the Portsmouth mounting.

Case 56. System 4 used. Foundation damping with Thiokol R. D. was inserted, and mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3,141.596$.

Case 57. System 4 used. Foundation damping with Thiokol R. D. was inserted, mount stiffness was decreased by setting $\omega_0 = 50.266$ and $\omega_t = 3,141.596$, and mount damping was increased by setting $Q_0 = 2.5$.

A-6 Sample Calculations

As noted in Section 2.5.2, the computational effort involved in the thesis was so great as to render it impossible without the use of a digital computer. However, the computational effort was largely needed in iterative calculations. To illustrate the complexity and length of the calculations involved in solving this problem, a sample calculation for one value of transmission is presented here.

For example, Case 1 properties are used with the driving frequency at 10.3 cps or 65 rad/sec.

The unsimplified four pole matrix equation for Case 1 follows:

$$\begin{bmatrix} F_1 \\ \\ V_1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{1}{M(\frac{L}{2})} \\ \\ 0 & 1 \end{bmatrix}}_{\text{Machine}} \underbrace{\begin{bmatrix} 1 & 0 \\ \\ \frac{1}{Z_M} & 1 \end{bmatrix}}_{\text{Mounting}} \underbrace{\begin{bmatrix} 1 + \frac{M_c}{M(L)} & \frac{1}{M(L)} \\ \\ M_c & 1 \end{bmatrix}}_{\text{Foundation}}$$

$$\underbrace{\begin{bmatrix} 1 & Z_H \\ \\ 0 & 1 \end{bmatrix}}_{\text{Hull}} \underbrace{\begin{bmatrix} 1 & 0 \\ \\ \frac{1}{R} & 1 \end{bmatrix}}_{\text{Water}} \begin{bmatrix} F_6 \\ \\ V_6 \end{bmatrix} \quad (1)$$

The following are the properties for this case:

1. Machine

   $W_B = 50$ lbf

   $m_B = .1295$ lbm

   $L = 60.0$ in

   $t = .75$ in

2. Mounting

   $f_0 = 16$ cps

   $Q_0 = 7.0$

   $f_t = 1000$ cps

   $W = 50.0$

3. Foundation

   $W_B = 30.0$ lbf

   $m_B = .078$ lbm

   $L = 36.0$ in

   $t = .75$ in

4. Column

$L = 6.0$ in

$m_c = .039$ lbm

$W_c = 15.0$ lbf

5. Hull

The nine parameters are

$R_s = 50$ lbf-sec/in

$R_1 = .01$ lbf-sec/in

$R_p = 5000$ lbf-sec/in

$X_s = 500$ lbf-sec/in

$W_s = 10.0$ lbf

$K_s = 10^6$ lbf/in

$W_p = 15$ lbf

$K_p = 2 \times 10^6$ lbf/in

6. Water

$R = 37{,}600$ lbf-sec/in

These properties are also listed in Appendix A-5.2.

The individual four pole parameters must be evaluated before combination of the matrices.

Machine:

From Appendix A-4.1:

$$M\left(\frac{L}{2}\right) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n^2\left(\frac{L}{2}\right)}{\omega_n^2 - \omega^2} - \frac{j}{m_B\omega}$$

where:

$$\omega_n = \lambda_n \sqrt{\frac{EI}{m_B}}$$

$$\phi_n\left(\frac{L}{2}\right) = \cosh\frac{\beta_n L}{2} + \cos\frac{\beta_n L}{2} - \left[\frac{\sinh\beta_n L + \sin\beta_n L}{\cosh\beta_n L - \cos\beta_n L}\right]\left[\sinh\frac{\beta_n L}{2} + \sin\frac{\beta_n L}{2}\right]$$

λ_n and $\phi_n(\frac{L}{2})$ are listed in Reference (16). ω_n is calculated.

$\omega_1 = 273$ $\quad\omega_6 = 3620$ $\quad\phi_1 = -1.21565$ $\quad\phi_6 = 0$

$\omega_2 = 751$ $\quad\omega_7 = 5080$ $\quad\phi_2 = 0$ $\quad\phi_7 = -1.41412$

$\omega_3 = 1475$ $\quad\omega_8 = 6780$ $\quad\phi_3 = +1.42238$ $\quad\phi_8 = 0$

$\omega_4 = 2440$ $\quad\omega_9 = 8730$ $\quad\phi_4 = 0$ $\quad\phi_9 = +1.41412$

$\omega_5 = 3630$ $\quad\omega_{10} = 10900$ $\quad\phi_5 = -1.41386$ $\quad\phi_{10} = 0$

For $\omega = 65$, using the above and listed properties

$$M\left(\frac{L}{2}\right) = \frac{j(65)}{.1295} \sum_{n=1}^{10} \frac{\phi_n^2(\frac{L}{2})}{\omega_n^2 - 4225} - \frac{j}{(65)(.1295)}$$

$$M\left(\frac{L}{2}\right) = -\ .1074\ j$$

$$\frac{1}{M(\frac{L}{2})} = 9.30\ j$$

Mounting:

From Appendix A-4.2:

$$\frac{1}{Z_M} = \frac{\omega Q_0}{K}\left[\frac{1 + jQ_0}{Q_0^2 + 1}\right]$$

$$K = \frac{W\omega_0^2\left[1 + \frac{\pi\omega}{t}\right]}{g\left[1 + \pi\frac{\omega_0}{\omega_t}\right]}$$

For $\omega = 65$:

$$K = \frac{(.1295)(100.53)^2 \left[1 + \frac{\pi 65}{6283.2}\right]}{1 + \frac{\pi(100.53)}{6283.2}}$$

$$K \doteq 1285$$

$$\frac{1}{Z_M} = \frac{(65)(7)}{1285} \left[\frac{1 + 7j}{49 + 1}\right]$$

$$\frac{1}{Z_M} = .00708 + .0496j$$

Foundation:

From Appendix A-4.3:

$$M(L) = \frac{j\omega}{m_B} \sum_{n=1}^{\infty} \frac{\phi_n(\frac{L}{2})\,\phi_n(L)}{\omega_n^2 - \omega^2} - \frac{j}{m_B \omega}$$

The values of ω_n and $\phi_n(L)$ were calculated as for the machine. The values of $\phi(\frac{L}{2})$ are the same as for the machine.

$\omega_1 = 758$	$\omega_6 = 10100$	$\phi_1 = +2.0$	$\phi_6 = -2.0$
$\omega_2 = 2085$	$\omega_7 = 14080$	$\phi_2 = -2.0$	$\phi_7 = +2.0$
$\omega_3 = 4090$	$\omega_8 = 18780$	$\phi_3 = +2.0$	$\phi_8 = -2.0$
$\omega_4 = 6770$	$\omega_9 = 24200$	$\phi_4 = -2.0$	$\phi_9 = +2.0$
$\omega_5 = 10100$	$\omega_{10} = 30100$	$\phi_5 = +2.0$	$\phi_{10} = -2.0$

For $\omega = 65$:

$$M(L) = \frac{j(65)}{(.078)} \sum_{n=1}^{10} \frac{\phi_n(\frac{L}{2})\,\phi_n(L)}{\omega_n^2 - 4225} - \frac{j}{(65)(.078)}$$

$$M(L) = -\ .2008\ j$$

$$\frac{1}{M(L)} = 4.98\ j$$

$$M_c = \frac{j\omega}{m_c} \sum_{n=1}^{\infty} \frac{2(-1)^{\frac{n-1}{2}}}{(\omega_n^2 - \omega^2)} - \frac{j}{m_c \omega}$$

From Reference (1):

$$\omega_n = \frac{n\pi}{L} \sqrt{\frac{Eg}{\gamma}}$$

$$\omega_1 = 3.33 \times 10^5$$

$$\omega_2 = 9.99 \times 10^5$$

$$\omega_3 = 16.65 \times 10^5$$

$$\omega_4 = 23.31 \times 10^5$$

$$\omega_5 = 29.97 \times 10^5$$

For $\omega = 65$:

$$M_c = \frac{j(65)}{(.039)} \sum_{n=1}^{5} \frac{2(-1)^{\frac{n-1}{2}}}{(\omega_n^2 - 4225)} - \frac{j}{(.039)(65)}$$

$$M_c = - .393\ j$$

Hull:

From Appendix A-4.4

$$Z = R + jX$$

$$R = R_s + \frac{R_1 \omega}{2\pi} + \frac{2\pi R_2}{\omega} + \frac{R_p}{1 + R_p^2 \left(\frac{g}{\omega W_p} - \frac{\omega}{K_p}\right)^2}$$

$$R = 50 + \frac{(.01)(65)}{2\pi} + \frac{2\pi(1000)}{65} + \frac{5000}{1 + (5000)^2 \left(\frac{386}{(65)(15)} - \frac{65}{2 \times 10^6}\right)^2}$$

$$R = 146.704$$

$$X = X_s + \frac{\omega W_s}{g} - \frac{K_s}{\omega} + \frac{R_p^2\left(\frac{g}{\omega W_p} - \frac{\omega}{K_p}\right)}{1 + R_p^2\left(\frac{g}{\omega W_p} - \frac{\omega}{K_p}\right)^2}$$

$$X = 500 + \frac{(65)(10)}{386} - \frac{10^6}{65} + \frac{(5000)^2\left(\frac{386}{(65)(15)} - \frac{65}{2 \times 10^6}\right)}{1 + (5000)^2\left(\frac{386}{(65)(15)} - \frac{65}{2 \times 10^6}\right)}$$

$$X = - 14{,}898.3$$

$$Z = 146.7 - 14{,}898.3\, j$$

Water:

From Appendix A-4.5

$$R = 37{,}600$$

$$\frac{1}{R} = .0000266$$

Substituting the computed four pole parameters in equation (1) in polar form:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & 9.30\angle\frac{\pi}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ .05\angle 1.428 & 1 \end{bmatrix} \begin{bmatrix} 2.96\angle 0 & 4.98\angle\frac{\pi}{2} \\ -.393\angle\frac{\pi}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 14{,}902\angle\frac{3\pi}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2.66 \times 10^{-5}\angle 0 & 1 \end{bmatrix} \begin{bmatrix} F_6 \\ V_6 \end{bmatrix} \quad (2)$$

This equation will reduce to the form:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix} \begin{bmatrix} F_6 \\ V_6 \end{bmatrix} \quad (3)$$

With reference to Figure V:

$$V_6 = 0 \quad (4)$$

From the four poles of the element representing the water:

$$F_5 = F_6 \tag{5}$$

Substituting equations (4) and (5) into equation (3):

$$\frac{F_1}{F_5} = \epsilon_{11}$$

In section 2.5.1 the transmission T was defined as:

$$T = 20 \log_{10} \frac{F_5}{F_1}$$

Therefore:

$$T = 20 \log_{10} \frac{1}{\epsilon_{11}}$$

From obtaining equation (3):

$$\epsilon_{11} = 5.51$$

At $\omega = 65$:

$$T = -14.85$$

A-7 <u>The IBM 709 Digital Computer Program</u>

A-7.1 Computer Logic

The logical procedure comprising the program written to solve this problem can best be illustrated by the computer flow diagram in Figure A-4.

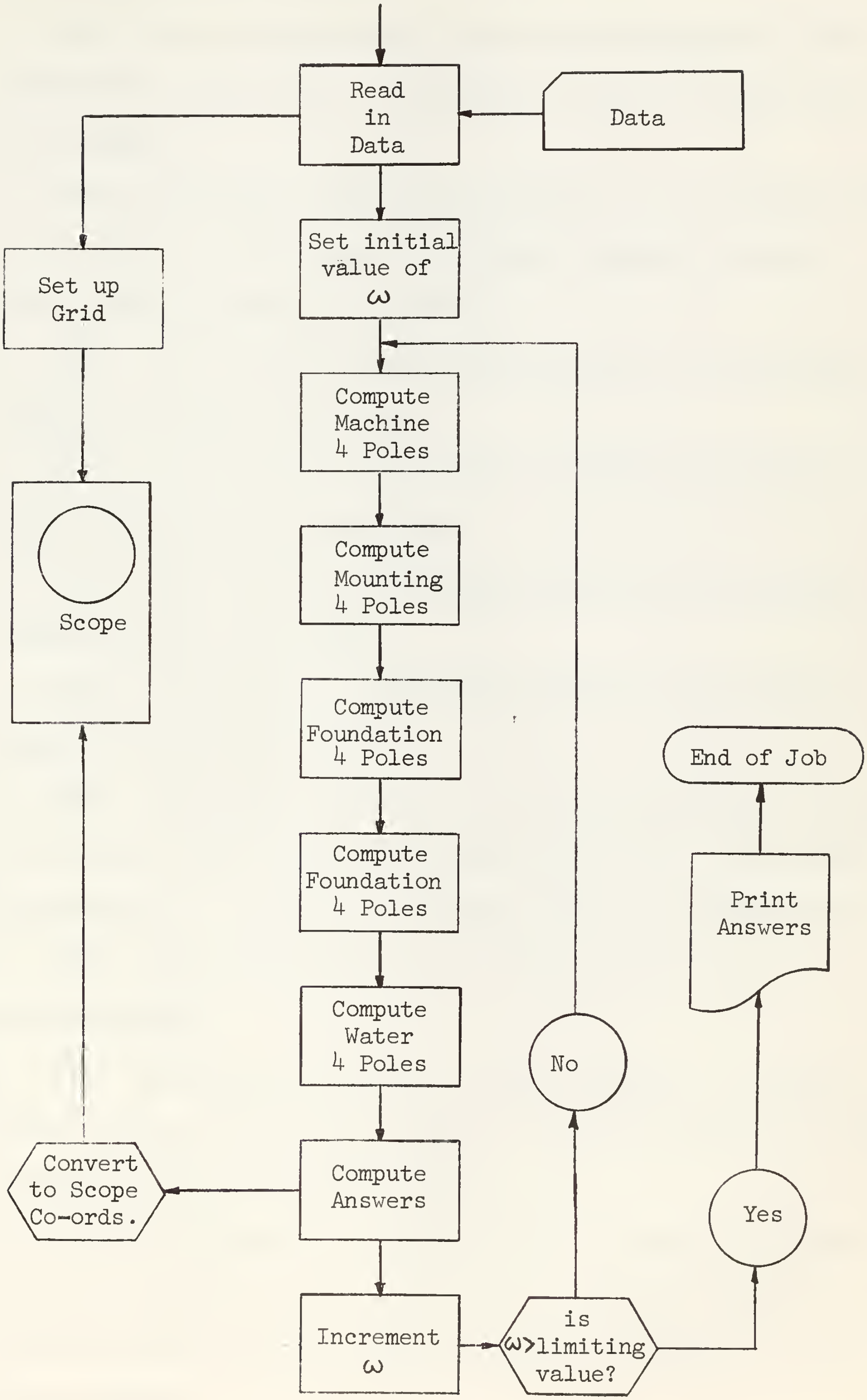
A-7.2 Scope

The cathode ray tube oscilloscope is a convenient computer feature which produces a graphical representation of the problem answers. The computer routine used to program this feature is not included in the program listing. Writtenin FAP, this routine projected a three cycle semi-log grid on the scope, labeled the grid, and plotted the answers. Photographs, 35 mm. in size, were taken of the scope presentations. These photographs appear as results in Appendix B.

A-7.3 Description of Subprograms

Figure A-4

Computer Logical Flow Diagram



1. MAIN This serves as the control program calling all subprograms when needed in the solution.

2. RDIN This subprogram reads in the data including the system properties. It also calls the routine to place the semi-log grid on the scope.

3. MAT 1 This program computes the four poles for the first element in the system. In this case the four pole parameters for the machine representation were obtained.

4. MAT 2 This serves the same purpose as MAT 1 for the second element or mounting in this case.

5. MAT 3 This serves the same purpose as MAT 1 for the third element or foundation in this case.

6. MAT 4 This serves the same purpose as MAT 1 for the fourth element or hull in this case.

7. MAT 5 This serves the same purpose as MAT 1 for the fifth element or water in this case.

8. TRANS This program takes all the four pole parameters computed by the various MAT routines and stores them for use in the matrix multiplication. It sets up the matrices prior to multiplication.

9. MAMUL This program reduces the complex matrices to one resultant matrix.

10. ANSW This program selects the matrix elements to obtain the answer from the final matrix of MAMUL. It also provides the routine for plotting the points on the scope.

11. CHOMEG This program tests the value of frequency and transmission to insure that the scope points are numerous enough to give an adequate plot. It also changes ω before returning to the start of the program.

12. PRINO This program prints the values of frequency and transmission to conclude the program.

A-7.4 Program Listing

The computer program listing is included to facilitate its possible future use in design investigations.

```
C      MAIN PROGRAM
       DIMENSION A(25),B(25),C(20),D(20),E(400),G(400),G1(400),V(200),VB
      1(20),AA(200),BB(200),CC(200),DD(100),EE(100)
       CALL RDIN (A,B,C,D,AA,BB,CC,DD,EE)
       G(1) = A(9)
       DO 210 L=1,400
C      MATRIX CALL SEQUENCE
       CALL MAT1 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       CALL MAT2 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       CALL MAT3 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       CALL MAT4 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       CALL MAT5 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
C      TRANSLATION SEQUENCE
       CALL TRANS (V,EE)
C      MATRIX MULTIPLICATION
       CALL MAMUL (I,J,K,V,A)
C      ANSWERS
       CALL ANSW (A,G1,G,E,L,V)
C      CALL FREQUENCY CHANGE
       CALL CHOMEG (E,L,G,A)
       IF(G(L)-A(8)) 210,240,240
 210   CONTINUE
 240   CALL PRINO (G1,G,E,L)
       CALL EXIT
       END

       SUBROUTINE TELE
       CALL FRAME
       CALL GRID
       CALL GRID
       CALL GRID
       CALL GRID
       CALL GRID
       CALL GRID
       CALL TITLE
       CALL VERT
       CALL HOR
       RETURN
       END

       SUBROUTINE RDIN (A,B,C,D,AA,BB,CC,DD,EE)
       DIMENSION A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),DD(100),
      1EE(100)
       READ 213,((A(I),I=1,10),(B(I),I=1,20),(C(I),I=1,20),(D(I),I=1,20),
      1(DD(I),I=1,10))
 213   FORMAT (5E13.7)
       CALL TELE
       RETURN
       END

       SUBROUTINE MAT1 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
C      BEAM MATRIX
       DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
      1DD(100),EE(100),VB(20)
       V43=1.0/(B(2)*G(L))
```

```
      DO 205 I=1, 10
      IF(D(I)**2-G(L)**2) 201,202,201
  201 VB(I)=G(L)*C(I)**2/(D(I)**2-G(L)**2)
      GO TO 205
  202 IF (G(L)*C(I)**2-0.0) 204,203,204
  203 VB(I)=0.0
      GO TO 205
  204 VB(I)=A(7)
  205 CONTINUE
      V78=VB(1)+VB(2)+VB(3)+VB(4)+VB(5)+VB(6)+VB(7)+VB(8)+VB(9)+VB(10)
      V79=V78/B(2)
      V229=V79-V43
      IF(V229-0.0) 206,208,207
  206 V329=A(6)
      GO TO 209
  207 V329=A(2)
      GO TO 209
  208 V329=0.0
      V328=A(7)
      GO TO 212
  209 V328=1.0/ABSF(V229)
  212 EE(1) = V328
      EE(2) = V329
      RETURN
      END

C     THIS IS MOUNT
      SUBROUTINE MAT2 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100)
      VA = (B(4)**2*B(5))*((B(6)+G(L)*A(1))/(B(6)+B(4)*A(1)))
      RE = (G(L)*B(3))/(VA*(B(3)**2+1.0))
      VE = RE*B(3)
      EE(3) = SQRTF(VE**2+RE**2)
      EE(4) = ATANF(VE/RE)
      RETURN
      END

      SUBROUTINE MAT2(G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100)
      GEE = 72.6*EXPF(0.4*LOGF(G(L)))
      RAR = 0.1*EXPF(0.325*LOGF(G(L)))
      V43 = RAR**2
      IF(V43-1.0) 410,411,410
  410 RE = (B(19)/B(20))*(GEE/G(L))*(2.0*RAR/(1.0-V43))
      VA = (B(19)/B(20))*(GEE/G(L))
      EE(3) = 1.0/SQRTF(RE**2+VA**2)
      EE(4) = ATANF(VA/RE)
      GO TO 412
  411 EE(3) = 0.0
      EE(4) = 0.0
  412 RETURN
      END
```

```
      SUBROUTINE MAT2(G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100)
      GEE = 72.6*EXPF(0.4*LOGF(G(L)))
      RAR = 0.1*EXPF(0.325*LOGF(G(L)))
      V43 = RAR**2
      IF(V43-1.0) 410,411,410
  410 RR = (B(19)/B(20))*(GEE/G(L))*(2.0*RAR/(1.0-V43))
      AA = (B(19)/B(20))*(GEE/G(L))
      GO TO 412
  411 RR = A(7)
  412 VA = (B(4)**2*B(5))*((B(6)+G(L)*A(1))/(B(6)+B(4)*A(1)))
      RE = (G(L)*B(3))/(VA*(B(3)**2+1.0))
      VE = RE*B(3)
      V47 = SQRTF(RR**2+AA**2)
      V48 = ATANF(AA/RR)
      V49 = SQRTF(RE**2+VE**2)
      V50 = ATANF(VE/RE)
      TOP1 = V47*V49
      TOP2 = V48+V50
      BOT1 = SQRTF((RE+RR)**2+(AA+VE)**2)
      BOT2 = ATANF((AA+VE)/(RE+RR))
      EE(3) = TOP1/BOT1
      EE(4) = TOP2-BOT2
      RETURN
      END

C     THIS IS FOUNDATION
C     MATRIX THREE
      SUBROUTINE MAT3 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100),VB(20)
      V43 = 1.0/(B(7)*G(L))
      DO 305 I = 1,10
      IF(D(I+10)**2-G(L)**2) 301,302,301
  301 VB(I+10) = (G(L)*C(I)*C(I+10))/(D(I+10)**2-G(L)**2)
      GO TO 305
  302 IF (G(L)*C(I)*C(I+10)-0.0) 304,303,304
  303 VB(I+10) = 0.0
      GO TO 305
  304 VB(I+10) = A(7)
  305 CONTINUE
      V78 = VB(11)+VB(12)+VB(13)+VB(14)+VB(15)+VB(16)+VB(17)+VB(18)+
     1VB(19)+VB(20)
      V79 = V78/B(7)
      V229 = V79-V43
      IF(V229-0.0) 306,308,307
  306 EE(6) = A(6)
      GO TO 309
  307 EE(6) = A(2)
      GO TO 309
  308 EE(6) = 0.0
      EE(5) = A(7)
      GO TO 312
  309 EE(5) = 1.0/ABSF(V229)
```

```
 312   RETURN
       END
C      THIS IS FOR FOUNDATION WITH DAMPER FOLLOWING
C      MATRIX THREE
       SUBROUTINE MAT3 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
      1DD(100),EE(100),VB(20)
       V43 = 1.0/(B(7)*G(L))
       DO 305 I = 1,10
       IF(D(I+10)**2-G(L)**2) 301,302,301
 301   VB(I+10) = (G(L)*C(I)*C(I+10))/(D(I+10)**2-G(L)**2)
       GO TO 305
 302   IF (G(L)*C(I)*C(I+10)-0.0) 304,303,304
 303   VB(I+10) = 0.0
       GO TO 305
 304   VB(I+10) = A(7)
 305   CONTINUE
       V78 = VB(11)+VB(12)+VB(13)+VB(14)+VB(15)+VB(16)+VB(17)+VB(18)+
      1VB(19)+VB(20)
       V79 = V78/B(7)
       V229 = V79-V43
       IF(V229-0.0) 306,308,307
 306   EE(6) = A(6)
       GO TO 309
 307   EE(6) = A(2)
       GO TO 309
 308   EE(6) = 0.0
       EE(5) = A(7)
       GO TO 312
 309   EE(5) = 1.0/ABSF(V229)
 312   EE(16) = 1.0/A(10)
       EE(14) = 1.0+(EE(5)*EE(16))
       VA = (EE(5)*EE(16))*SINF(EE(6))
       RE = 1.0+(EE(5)*EE(16))*COSF(EE(6))
       EE(15) = ATANF(VA/RE)
       RETURN
       END
C      FOUNDATION WITH DAMPER VARIABLE WITH FREQUENCY
C      MATRIX THREE
       SUBROUTINE MAT3 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
       DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
      1DD(100),EE(100),VB(20)
       V43 = 1.0/(B(7)*G(L))
       DO 305 I = 1,10
       IF(D(I+10)**2-G(L)**2) 301,302,301
 301   VB(I+10) = (G(L)*C(I)*C(I+10))/(D(I+10)**2-G(L)**2)
       GO TO 305
 302   IF (G(L)*C(I)*C(I+10)-0.0) 304,303,304
 303   VB(I+10) = 0.0
       GO TO 305
 304   VB(I+10) = A(7)
 305   CONTINUE
       V78 = VB(11)+VB(12)+VB(13)+VB(14)+VB(15)+VB(16)+VB(17)+VB(18)+
```

```
     1VB(19)+VB(20)
      V79 = V78/B(7)
      V229 = V79-V43
      IF(V229-0.0) 306,308,307
 306  EE(6) = A(6)
      GO TO 309
 307  EE(6) = A(2)
      GO TO 309
 308  EE(6) = 0.0
      EE(5) = A(7)
      GO TO 312
 309  EE(5) = 1.0/ABSF(V229)
 312  GEE = 72.6*EXPF(0.4*LOGF(G(L)))
      RAR = 0.1*EXPF(0.325*LOGF(G(L)))
      V44 = RAR**2
      IF(V44-1.0)500,501,500
 500  RE = B(1)/A(10)*(GEE/G(L))*(2.0*RAR/(1.0-V44))
      EE(16) = 1.0/RE
      GO TO 502
 501  EE(16) = 0.0
 502  EE(14) = 1.0+(EE(5)*EE(16))
      VA = (EE(5)*EE(16))*SINF(EE(6))
      RR = 1.0+(EE(5)**EE(16))*COSF(EE(6))
      EE(15) = ATANF(VA/RR)
      RETURN
      END

C     THIS IS COLUMN HULL MATRIX 4
      SUBROUTINE MAT4 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100),VB(20)
      V43 = 1.0/(B(8)*G(L))
      DO 405 I = 1,5
      IF (DD(I)**2-G(L)**2) 401,402,401
 401  VB(I) = (G(L)*2.0*DD(I+5))/(B(8)*(DD(I)**2-G(L)**2))
      GO TO 405
 402  VB(I) = A(7)
 405  CONTINUE
      V78 = VB(1)+VB(3)+VB(5)+V43
      V79 = VB(2)+VB(4)
      EE(7) = SQRTF(V78**2+V79**2)
      IF(V79-0.0)407,406,407
 406  EE(8) = A(6)
 407  EE(8) = ATANF(V78/V79)
      V45 = B(9) +(B(11)*G(L))/A(3)+A(3)*B(12)/G(L)
      V46 = 386.0/B(17)*G(L)-G(L)/B(16)
      V45 = V45+B(15)/(1.0+B(15)**2*V46**2)
      V47 = B(10)+B(14)*G(L)/386.0-B(13)/G(L)+(B(15)**2*V46)/(1.0+B(15)*
     1*2*V46**2)
      EE(9) = SQRTF(V45**2+V47**2)
      EE(10) = ATANF(V47/V45)
      V50 = EE(7)*EE(9)
      V51 = EE(8)+EE(10)
      V52 = V50*COSF(V51)
      V53 = V50*SINF(V51)
```

```
      V52 = 1.0+V52
      EE(11) = SQRTF(V52**2+V53**2)
      EE(12) = ATANF(V53/V52)
      RETURN
      END

C     MATRIX FIVE
C     THIS IS WATER
      SUBROUTINE MAT5 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100)
      EE(13) = 1.0/B(18)
      RETURN
      END

      SUBROUTINE MAT5 (G,A,B,C,D,L,AA,BB,CC,DD,EE)
      DIMENSION G(400),A(25),B(25),C(20),D(20),AA(200),BB(200),CC(200),
     1DD(100),EE(100)
      EE(13) = G(L)**2/B(1)
      RETURN
      END

C     MATRIX MULTIPLICATION
      SUBROUTINE MAMUL (I,J,K,V,A)
      DIMENSION V(200),A(25)
      I=1
      J=9
      K=68
 101  K=K+2
      V41=V(I)*V(J)
      V42=V(I+1)+V(J+1)
      V43=SINF(V42)*V41
      V44=COSF(V42)*V41
      V45=V(I+2)*V(J+4)
      V46=V(I+3)+V(J+5)
      V47=SINF(V46)*V45
      V48=COSF(V46)*V45
      V49=V44+V48
      V50=V43+V47
      V(K)=SQRTF(V50**2+V49**2)
      IF(V44+V48) 50,25,50
 50   V(K+1) = ATANF(V50/V49)
      GO TO 26
 25   IF(V43+V47) 51, 27, 51
 51   V(K+1) = A(6)
      GO TO 26
 27   V(K+1) = 0.0
 26   IF(K-72) 1099, 102, 1100
 1100 IF(K-74) 1099, 3, 1101
 1101 IF(K-76) 1099,4,1102
 1102 IF(K-78) 1099,5,1103
 1103 IF(K-80) 1099,6,1104
 1104 IF(K-82) 1099,7,1105
 1105 IF(K-84) 1099,8,1106
 1106 IF(K-86) 1099,9,1107
```

```
1107 IF(K-88) 1099,10,1108
1108 IF(K-90) 1099,11,1109
1109 IF(K-92) 1099,12,1110
1110 IF(K-94) 1099,13,1111
1111 IF(K-96) 1099,14,1112
1112 IF(K-98) 1099,15,1113
1113 IF(K-100) 1099,16,1099
1099 I = I+16
     J = J+16
     GO TO 101
102  V102 = (V(70)*V(72))*SINF(V(71)+V(73))
     V103 = (V(70)*V(72))* COSF(V(71)+V(73))
     I = I-16
     J = J-14
     GO TO 101
3    I = I+20
     J = J+14
     GO TO 101
4    V104 = (V(74)*V(76))* SINF(V(75)+V(77))
     V105 = (V(74)*V(76))*COSF(V(75)+V(77))
     V(106) = V104+V102
     V(107) = V105+V103
     I = I-16
     J = J-16
     GO TO 101
5    I = I+12
     J = J+16
     GO TO 101
6    V102 = (V(78)*V(80))*SINF(V(79)+V(81))
     V103 = (V(78)*V(80))*COSF(V(79)+V(81))
     I = I-12
     J = J-14
     GO TO 101
7    I = I+16
     J = J+14
     GO TO 101
8    V104 = (V(82)*V(84))*SINF(V(83)+V(85))
     V105 = (V(82)*V(84))*COSF(V(83)+V(85))
     V(108) = V104 +V102
     V(109) = V105+V103
     I = I-20
     J = J-16
     GO TO 101
9    I = I+16
     J = J+18
     GO TO 101
10   V102 = (V(86)*V(88))*SINF(V(87)+V(89))
     V103 = (V(86)*V(88))*COSF(V(87)+V(89))
     I = I-16
     J = J-16
     GO TO 101
11   I = I+20
     J = J+16
     GO TO 101
12   V104=(V(90)*V(92))*SINF(V(91)+V(93))
```

```
      V105 = (V(90)*V(92))*COSF(V(91)+V(93))
      V(110) = V104 + V102
      V(111) = V105+ V103
      I = I-16
      J = J-18
      GO TO 101
13    I = I+12
      J = J+18
      GO TO 101
14    V102 = (V(94)*V(96))*SINF(V(95)+V(97))
      V103 = (V(94)*V(96))*COSF(V(95)+V(97))
      I = I- 12
      J = J-16
      GO TO 101
15    I = I+16
      J = J+16
      GO TO 101
16    V104 = (V(98)*V(100))*SINF(V(99)+V(101))
      V105 = (V(98)*V(100))*COSF(V(99)+V(101))
      V(112) = V104 +V102
      V(113) = V105+V103
      I = I+85
      J=  J+6
      K = K+20
17    K = K+2
      V43 = SQRTF(V(I)**2+V(I+1)**2)
      IF(V(I+1)+0.0) 52,28,52
52    V44 = ATANF(V(I)/V(I+1))
      GO TO 29
28    IF(V(I)+0.0) 53,30,53
53    V44 = A(6)
      GO TO 29
30    V44 = 0.0
29    V(K) = V43*V(J)*SINF(V44+V(J+1))
      V(K+1) = V43*V(J)*COSF(V44+V(J+1))
      IF(K-124)1098,18,1115
1115  IF(K-126)1098,19,1116
1116  IF(K-128)1098,20,1117
1117  IF(K-130)1098,21,1118
1118  IF(K-132)1098,22,1119
1119  IF(K-134)1098,23,1120
1120  IF(K-136)1098,24,1098
1098  I = I+4
      J = J+4
      GO TO 17
18    V138 = V(122)+V(124)
      V139 = V(123)+V(125)
      I = I-2
      J = J-4
      GO TO 17
19    I = I+4
      J = J+4
      GO TO 17
20    V140 = V(126)+V(128)
      V141 = V(127)+V(129)
```

```
      I=I-5
      J=J-2
      GO TO 17
21    I=I+4
      J=J+4
      GO TO 17
22    V142=V(130)+V(132)
      V143=V(131)+V(133)
      I=I-2
      J=J-4
      GO TO 17
23    I=I+4
      J=J+4
      GO TO 17
24    V144=V(134)+V(136)
      V145=V(135)+V(137)
      RETURN
      END

C     THIS IS TRANSLATION
      SUBROUTINE TRANS (V,EE)
      DIMENSION V(200),EE(100)
      V(1) = 1.0
      V(2) = 0.0
      V(3) = EE(1)
      V(4) = EE(2)
      V(5) = 0.0
      V(6) =0.0
      V(7) = 1.0
      V(8) = 0.0
      V(9) = 1.0
      V(10) = 0.0
      V(11) = 0.0
      V(12) = 0.0
      V(13) = EE(3)
      V(14) = EE(4)
      V(15) = 1.0
      V(16) = 0.0
      V(17) = 1.0
      V(18) = 0.0
      V(19) = EE(5)
      V(20) = EE(6)
      V(21) = 0.0
      V(22) = 0.0
      V(23) = 1.0
      V(24) = 0.0
      V(25) = 1.0
      V(26) = 0.0
      V(27) = EE(9)
      V(28) = EE(10)
      V(29) = EE(7)
      V(30) = EE(8)
      V(31) = EE(11)
      V(32) = EE(12)
      V(33) = 1.0
```

```
      V(34) = 0.0
      V(35) = 0.0
      V(36) = 0.0
      V(37) = EE(13)
      V(38) = 0.0
      V(39) = 1.0
      V(40) = 0.0
      RETURN
      END

*     FORTRAN
C     THIS IS TRANSLATION
C     THIS IS FOR FOUNDATION WITH DAMPER FOLLOWING
      SUBROUTINE TRANS (V,EE)
      DIMENSION V(200),EE(100)
      V(1) = 1.0
      V(2) = 0.0
      V(3) = EE(1)
      V(4) = EE(2)
      V(5) = 0.0
      V(6) =0.0
      V(7) = 1.0
      V(8) = 0.0
      V(9) = 1.0
      V(10) = 0.0
      V(11) = 0.0
      V(12) = 0.0
      V(13) = EE(3)
      V(14) = EE(4)
      V(15) = 1.0
      V(16) = 0.0
      V(17) = EE(14)
      V(18) = EE(15)
      V(19) = EE(5)
      V(20) = EE(6)
      V(21) = EE(16)
      V(22) = 0.0
      V(23) = 1.0
      V(24) = 0.0
      V(25) = 1.0
      V(26) = 0.0
      V(27) = EE(9)
      V(28) = EE(10)
      V(29) = EE(7)
      V(30) = EE(8)
      V(31) = EE(11)
      V(32) = EE(12)
      V(33) = 1.0
      V(34) = 0.0
      V(35) = 0.0
      V(36) = 0.0
      V(37) = EE(13)
      V(38) = 0.0
      V(39) = 1.0
      V(40) = 0.0
```

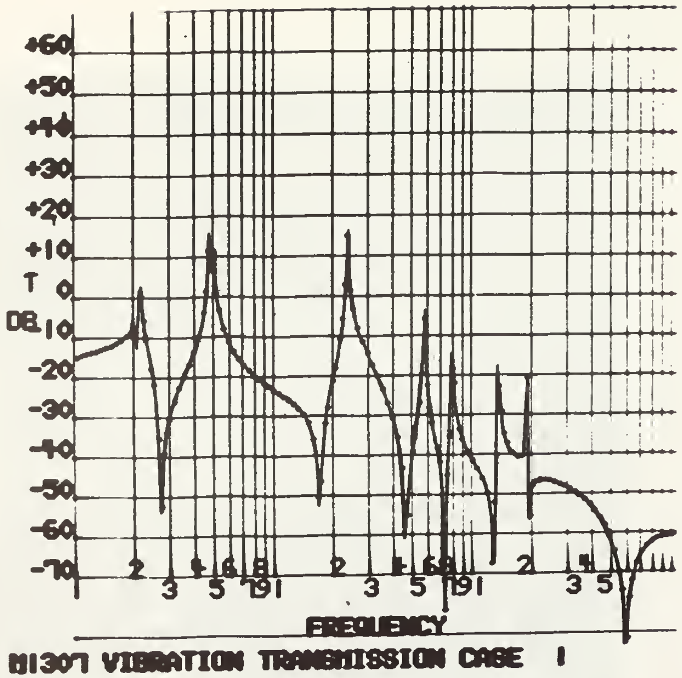
```
      RETURN
      END

      SUBROUTINE ANSW (A,G1,G,E,L,V)
      DIMENSION A(25),G1(400),G(400),E(400),V(200)
       V138 = V(122)+V(124)
      V139 = V(123)+V(125)
      E(L) = (A(5)/A(4))*LOGF(1.0/SQRTF(V138**2+V139**2))
      G1(L) = G(L)/A(3)
      G2 = LOGF(G1(L))/A(4)
      MFREQ = (((G2-1.0)*920.0)/3.0)+102.0
      MT = (((E(L)+70.0)*870.0)/140.0)+152.0
      CALL BRIPIC(MFREQ,MT)
      CALL BRIPIC (MFREQ,MT)
      RETURN
      END

      SUBROUTINE CHOMEG(E,L,G,A)
      DIMENSION E(400),G(400),A(25)
      IF(G(L)-A(9))251,250,251
251   V502 = ABSF(E(L)-E(L-1))
      V1012 = ABSF(LOGF(G(L))-LOGF(G(L-1)))
      IF(V502-5.0)254,253,253
254   IF(V502-2.5)250,255,255
255   IF(V1012-0.02878)250,253,250
253   G(L+1) = EXPF(LOGF(G(L))+.02878)
      GO TO 252
250   G(L+1) = EXPF(LOGF(G(L))+.05757)
252   RETURN
      END

      SUBROUTINE PRINO (G1,G,E,L)
      DIMENSION G1(400),G(400),E(400)
      PRINT 220, (G(I),G1(I),E(I),I=1,L)
220   FORMAT(8H OMEG = F8.1,9H  FREQ = F7.1,6H  T = E13.6)
      CALL FRAME
      RETURN
      END
```

APPENDIX B

PRESENTATION OF RESULTS

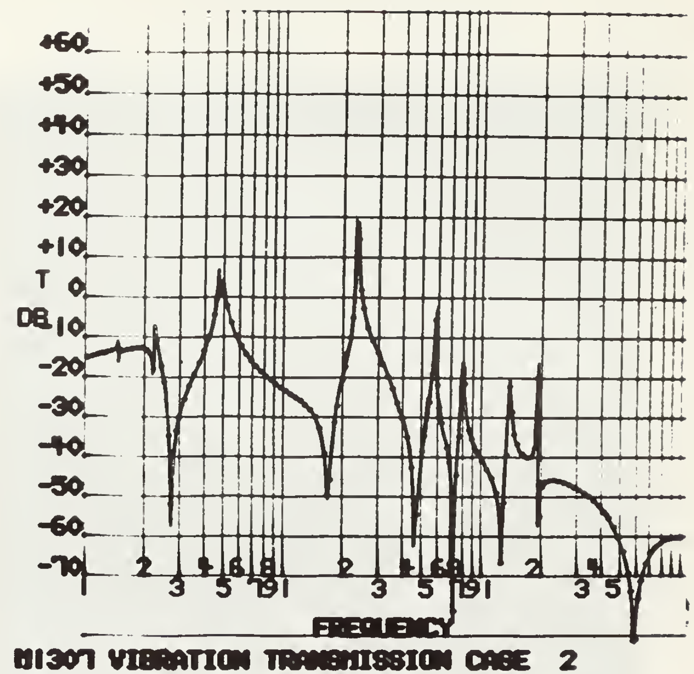
The following photo plots were produced by the Cathode Ray Tube Recorder of the IBM 709 Digital Computer. The computed points were connected by solid lines by the authors to clarify the plots.

Due to the space limitations of the scope presentation the abscissa or frequency scale is scaled in integers for three decades. The first decade on all of the plots extends from 10 to 100 cps, the second decade from 100 to 1000 cps, and the third from 1000 to 10,000 cps. The ordinate presents transmission in decibels varying from -70 to +60.



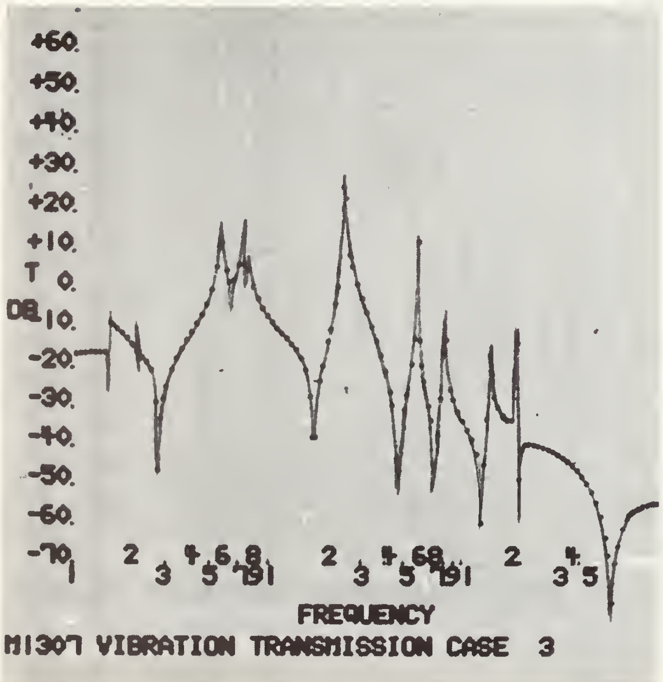
CASE 1

System 1 (Appendix A-5.2)
50 lb. Machine
30 lb. Foundation
Portsmouth Mounting



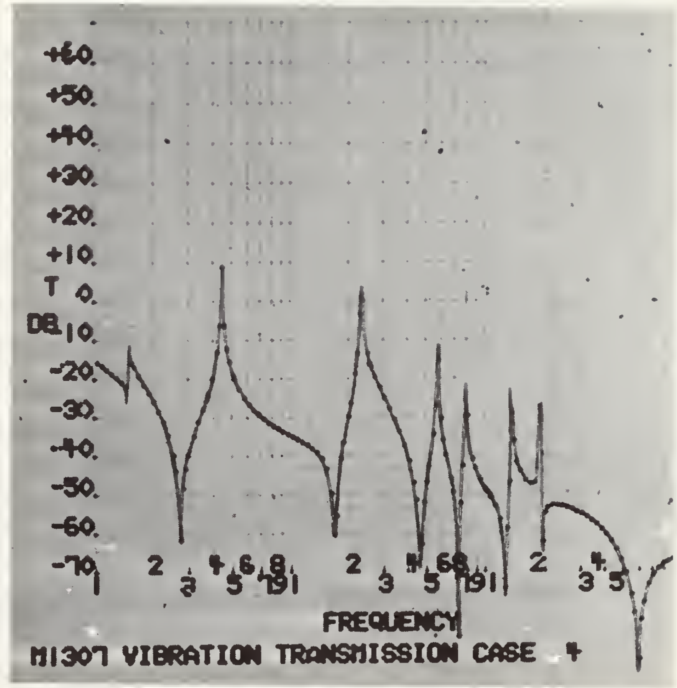
CASE 2

Same as Case 1, except
Increased Mount Damping



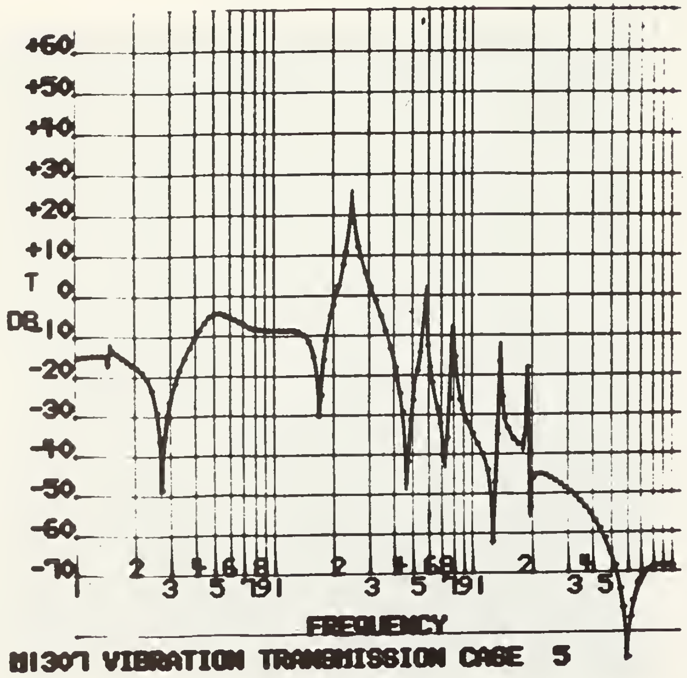
CASE 3

Same as Case 1, except
Increased Mount Stiffness



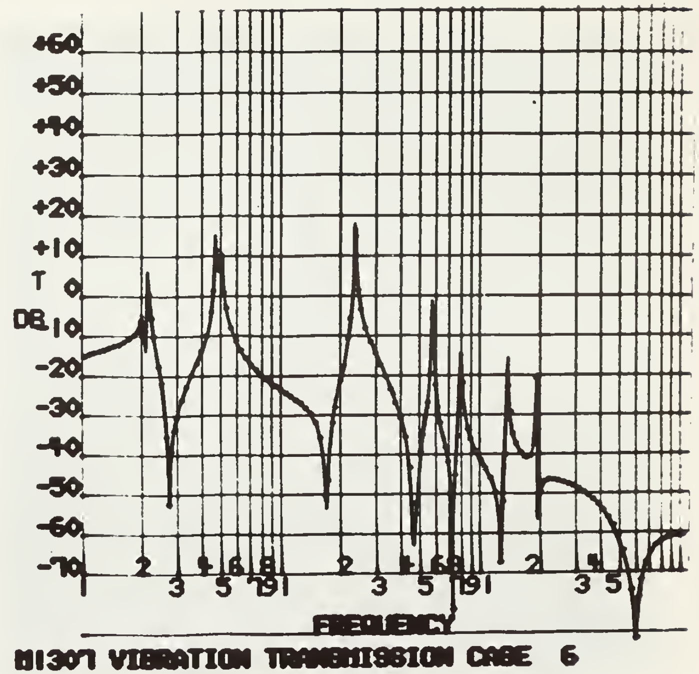
CASE 4

Same as Case 1, except
Decreased Mount Stiffness



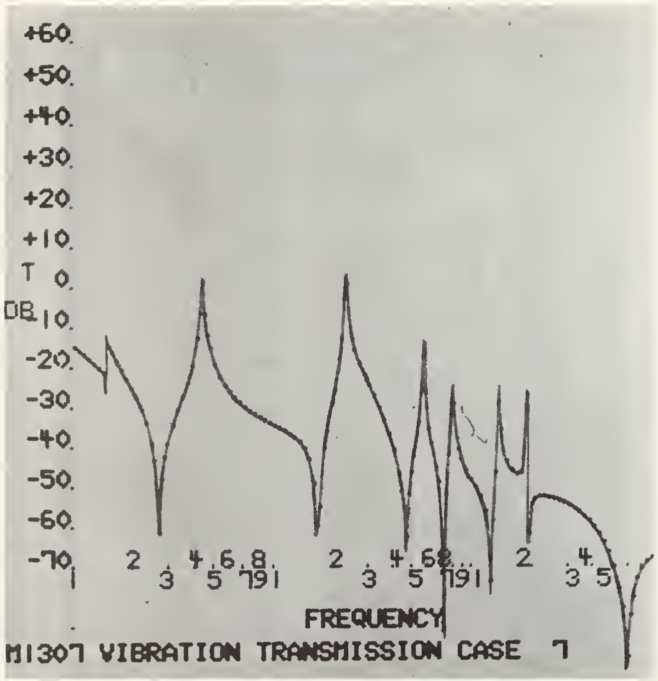
CASE 5

Same as Case 1, except
Thiokol R. D. Mount
Replaced Portsmouth Mount



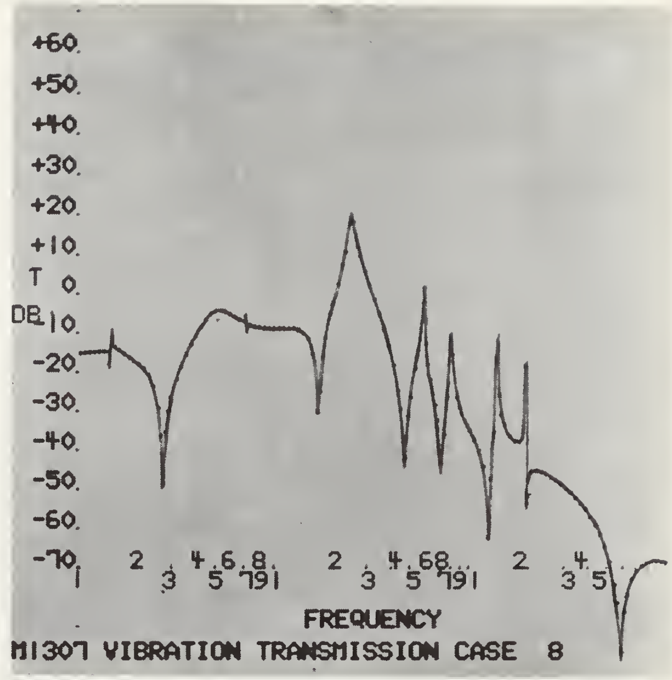
CASE 6

Same as Case 1, except
Increased Hull Damping, Doubled



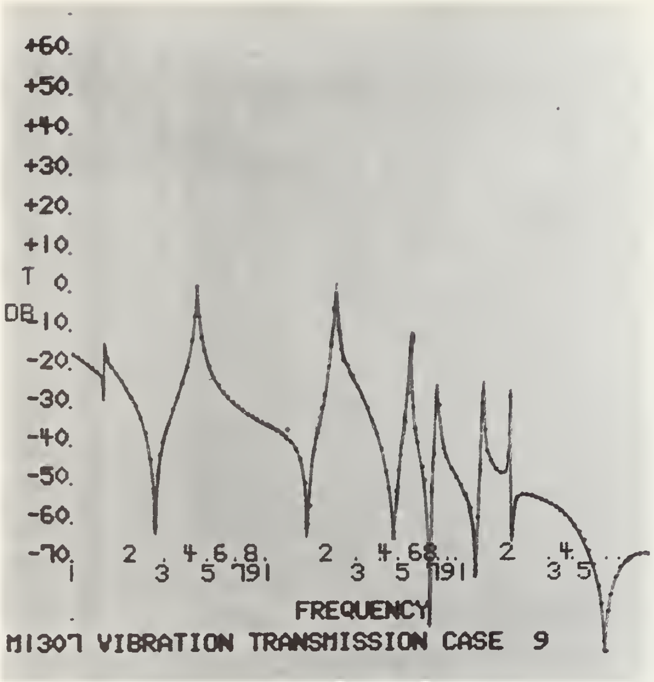
CASE 7

Same as Case 1, except
Increased Mount Damping,
Decreased Mount Stiffness



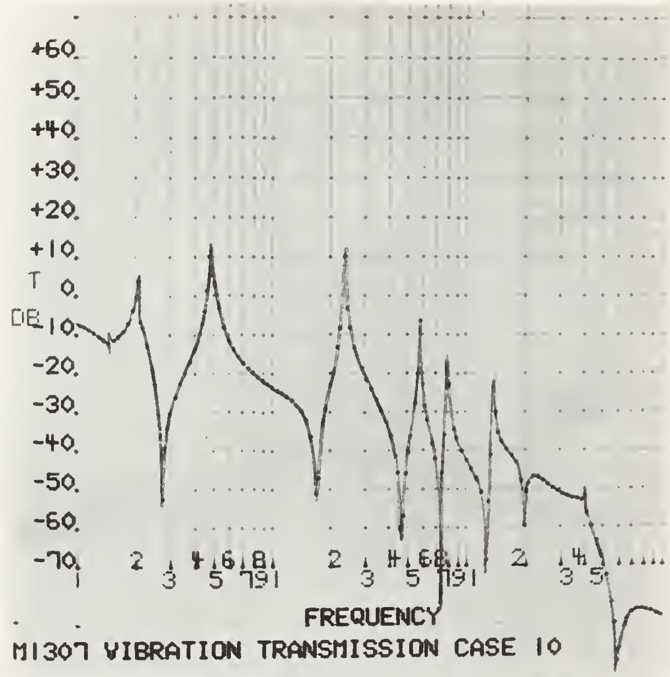
CASE 8

Same as Case 1, except
Thiokol R. D. Mount
Replaced Portsmouth Mount
Increased Hull Damping, Doubled



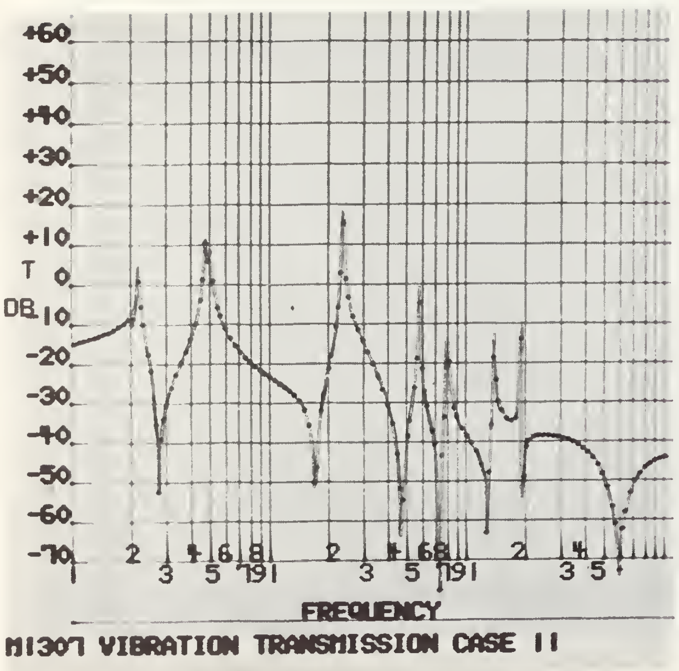
CASE 9

Same as Case 1, except
Increased Mount Damping
Decreased Mount Stiffness
Increased Hull Damping, 20 times original



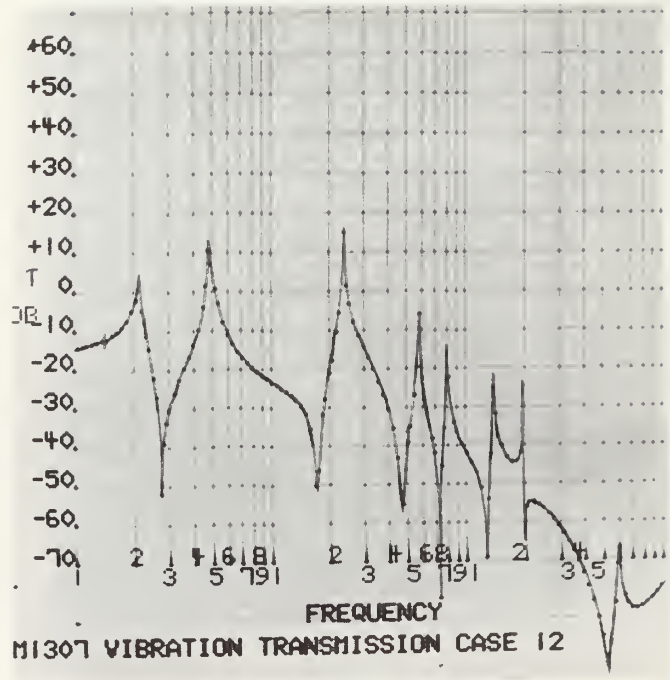
CASE 10

Same as Case 1, except
Foundation Damping, Viscous



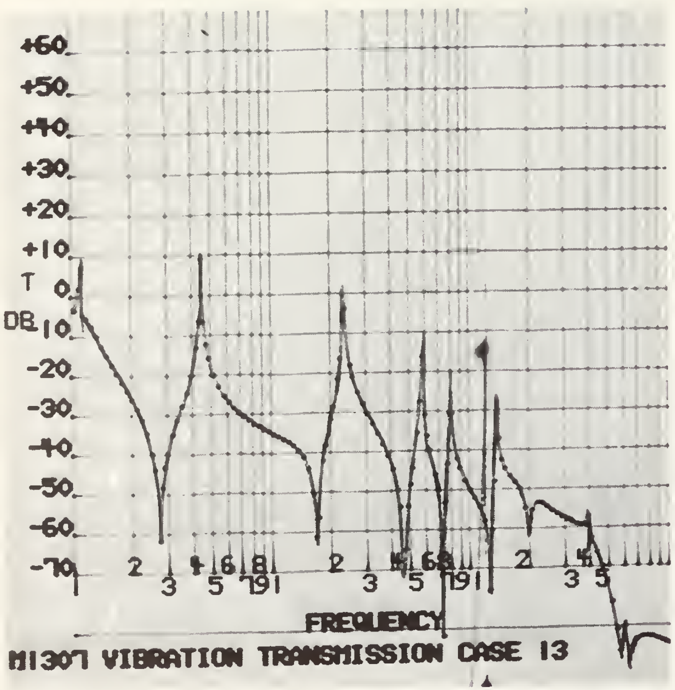
CASE 11

Same as Case 1, except
Thiokol R. D. and Portsmouth Mounts
In Parallel



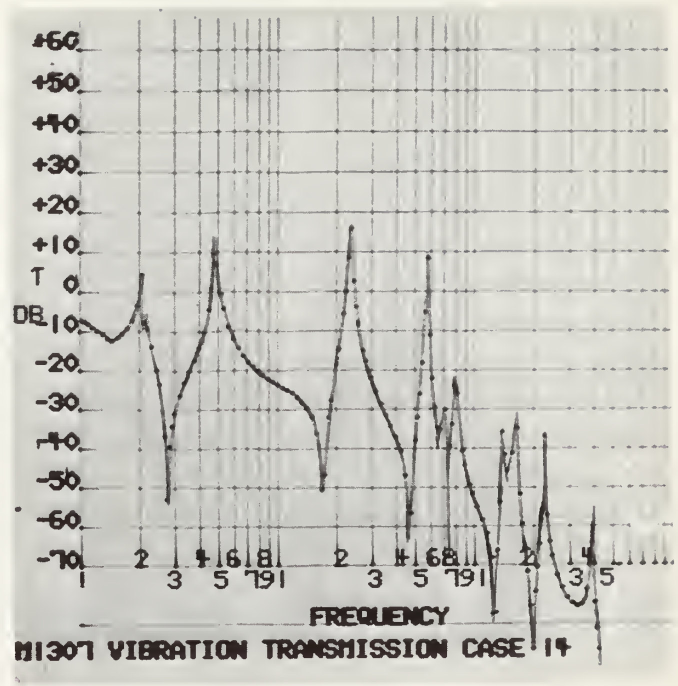
CASE 12

Same as Case 1, except
Water Resistance Varied
With Frequency



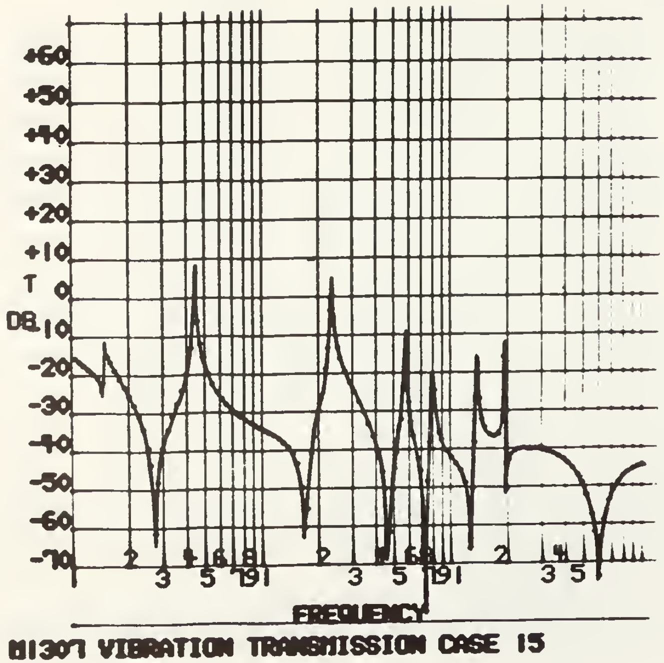
CASE 13

Same as Case 1, except
Decreased Mount Stiffness
Foundation Damping, Viscous



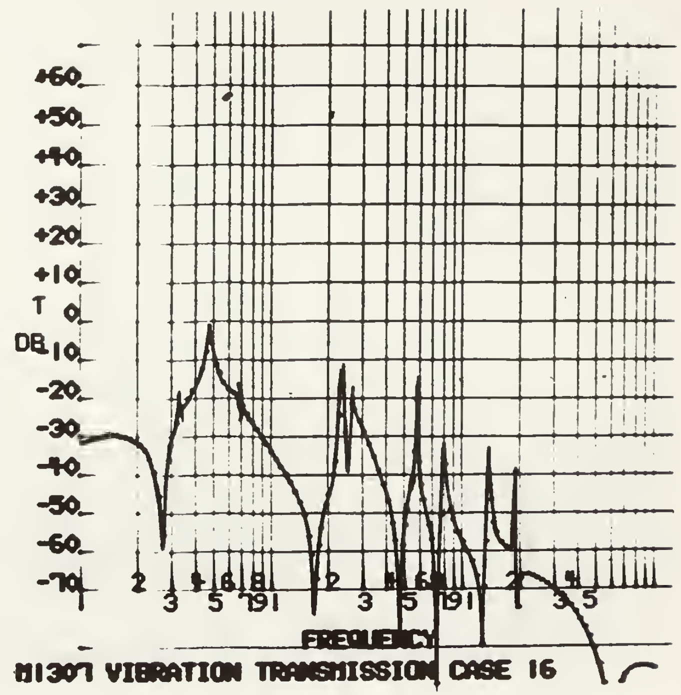
CASE 14

Same as Case 1, except
Foundation Damping, Thiokol R. D.



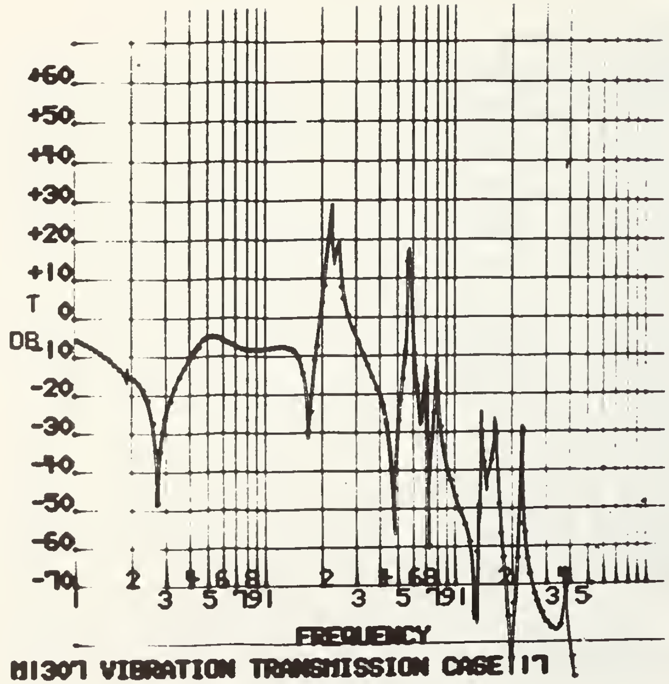
CASE 15

Same as Case 1, except
Thiokol R. D. and Portsmouth Mounts
In Parallel, but Decreased Stiffness
In Portsmouth Mount



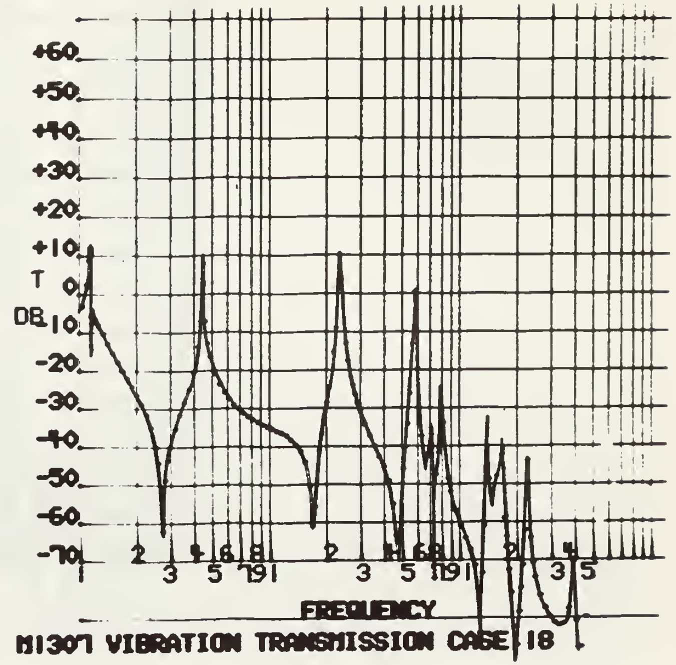
CASE 16

Same as Case 1, except
Increased Hull Damping, Maximum



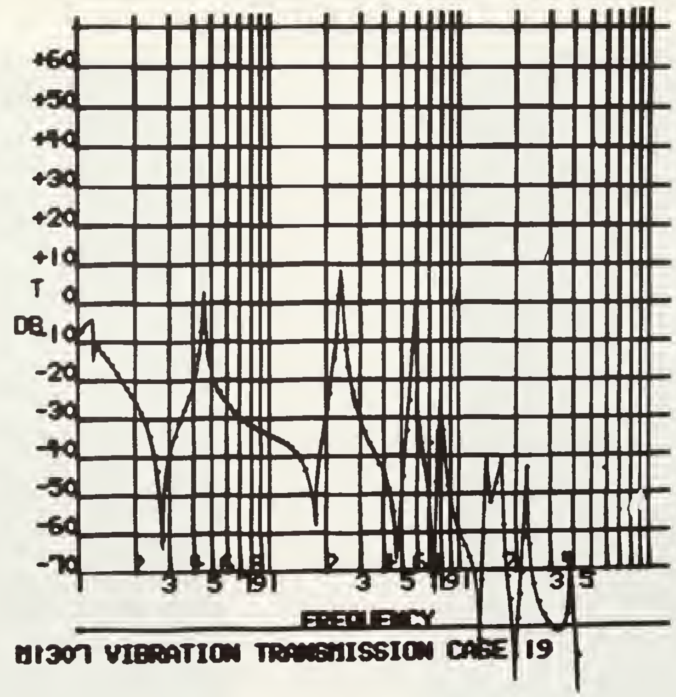
CASE 17

Same as Case 1, except
Thiokol R. D. Mount replaced
Portsmouth Mount, Foundation
Damping, Thiokol R. D.



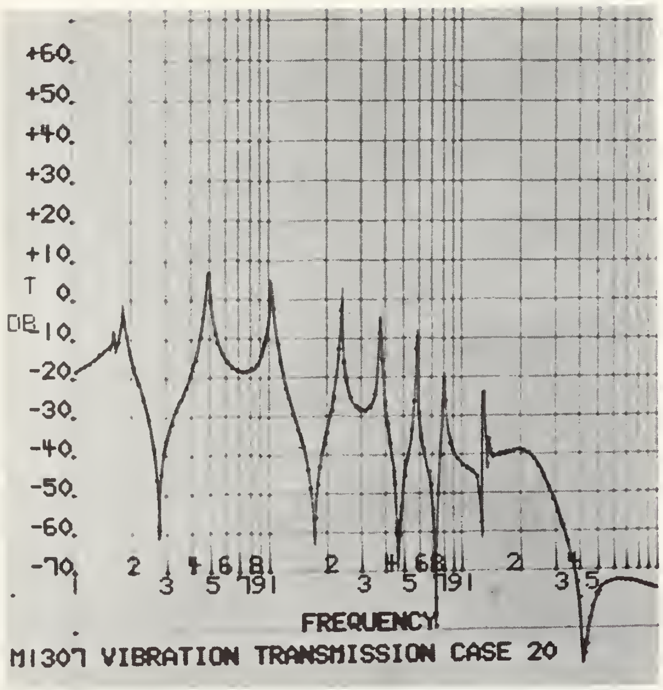
CASE 18

Same as Case 1, except
Foundation Damping, Thiokol R. D.
Decreased Mount Stiffness



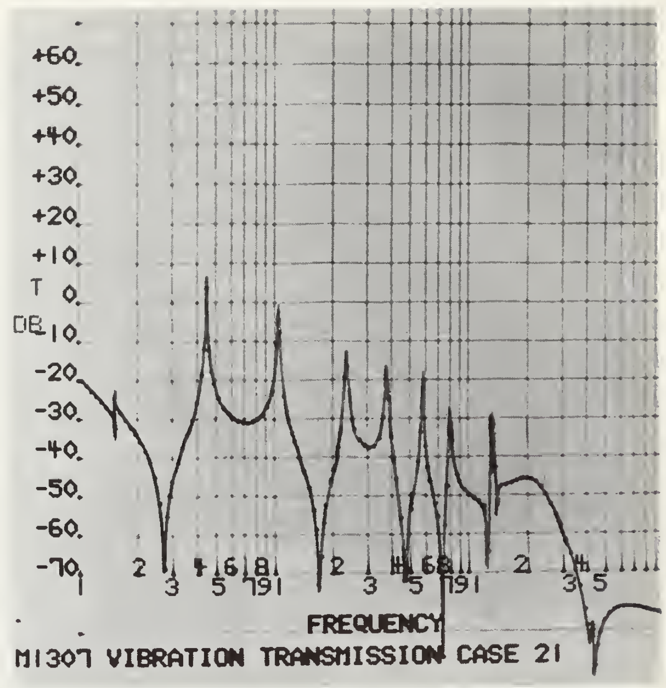
CASE 19

Same as Case 1, except
Foundation Damping, Thiokol R. D.
Decreased Mount Stiffness
Increased Mount Damping



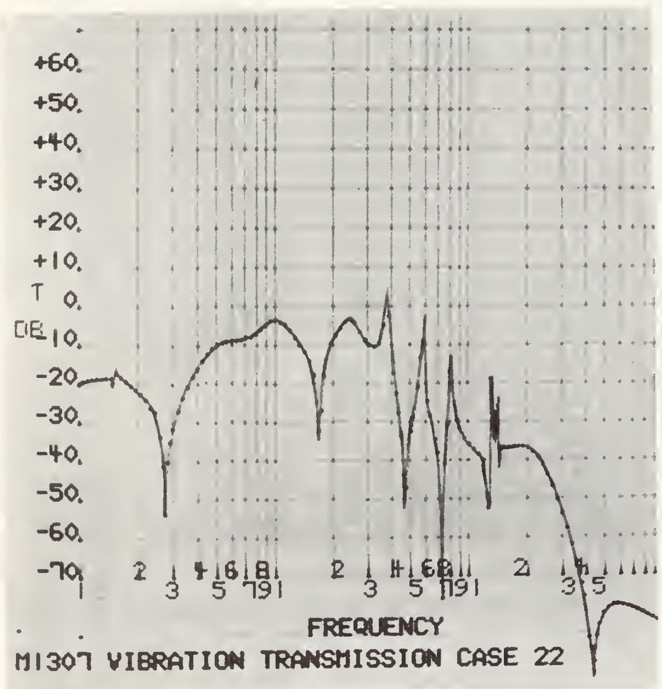
CASE 20

System 2 (Appendix A-5.3)
50 lb. Machine
100 lb. Foundation
Portsmouth Mounting



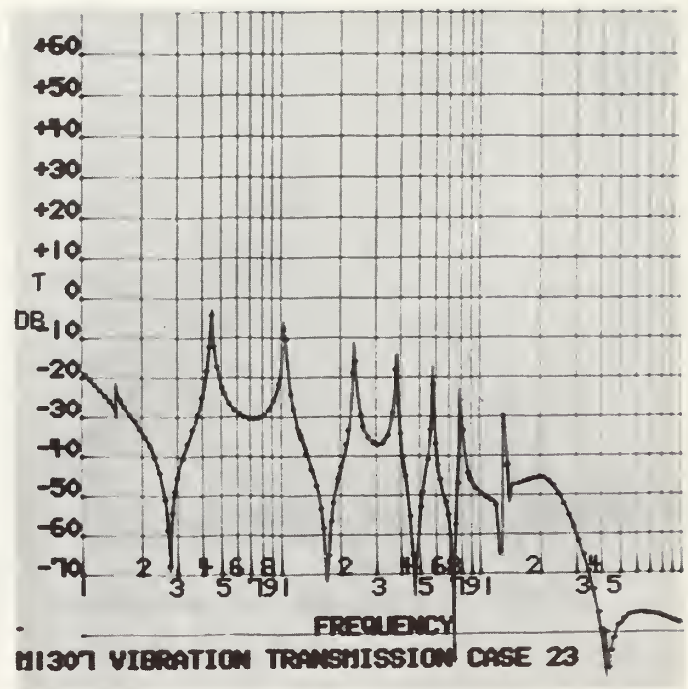
CASE 21

Same as Case 20, except
Decreased Mount Stiffness



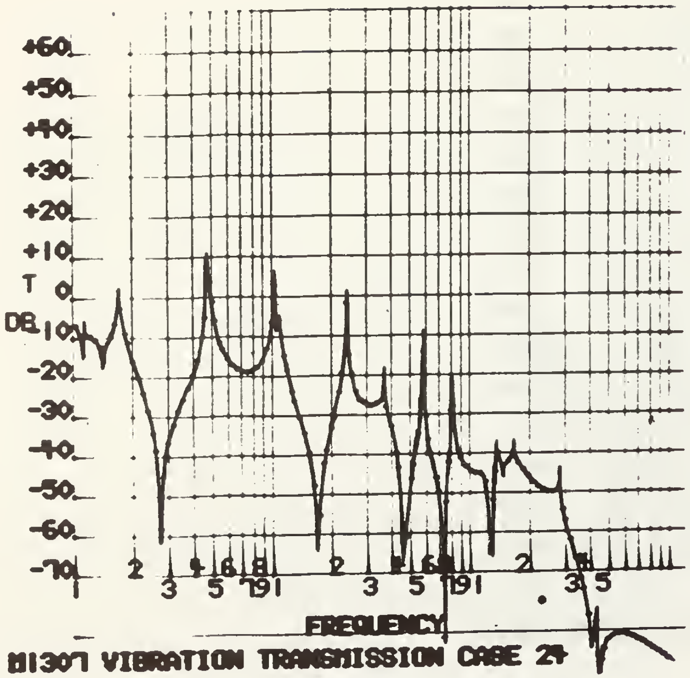
CASE 22

Same as Case 20, except
Thiokol R. D. Mount Replaced
Portsmouth Mount



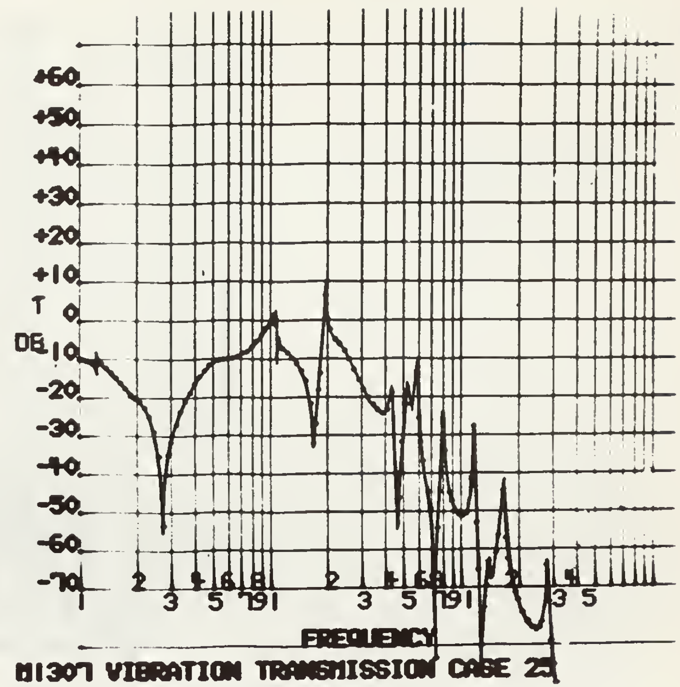
CASE 23

Same as Case 20, except
Increased Mount Damping
Decreased Mount Stiffness
Increased Hull Damping, 20 times



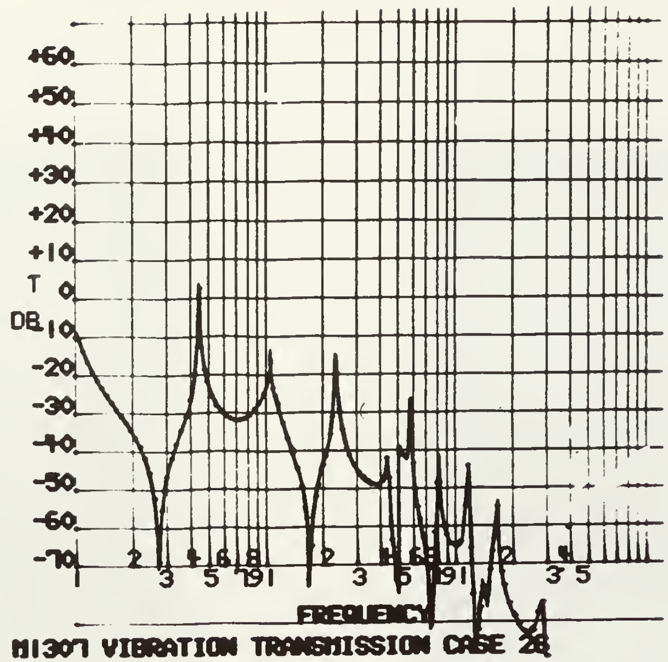
CASE 24

Same as Case 20, except
Foundation Damping, Viscous



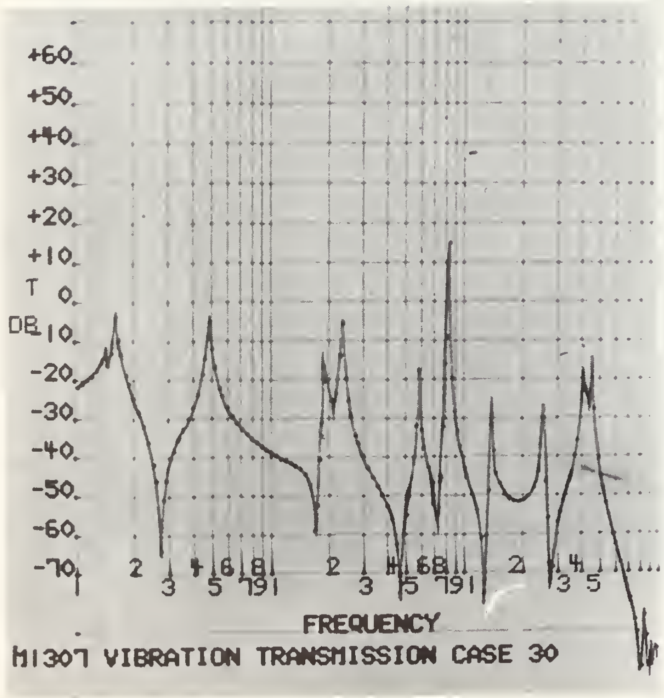
CASE 25

Same as Case 20, except
Thiokol R. D. Mount Replaced
Portsmouth Mount
Foundation Damping, Thiokol R. D.



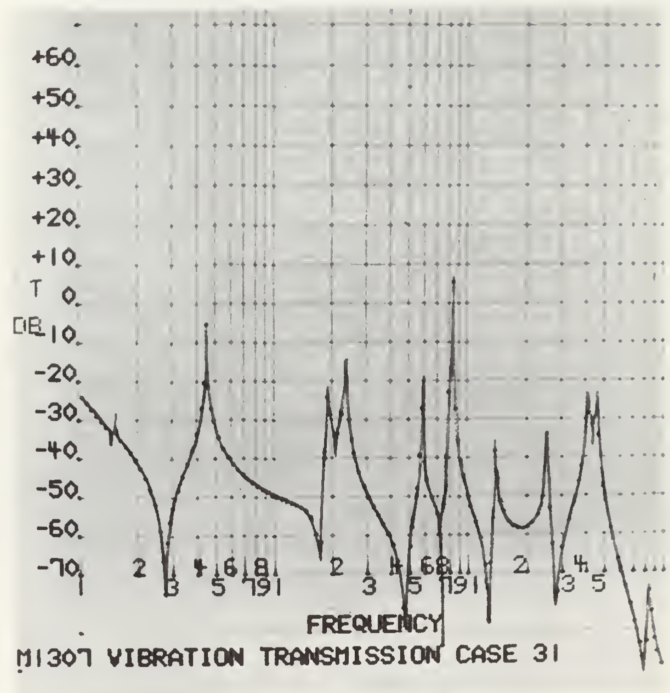
CASE 26

Same as Case 20, except
Foundation Damping, Thiokol R. D.
Decreased Mount Stiffness



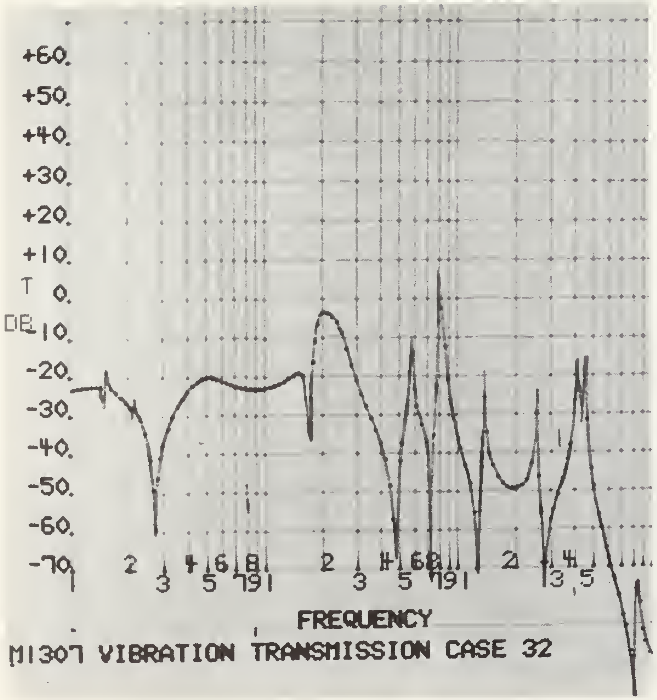
CASE 30

System 3 (Appendix A-5.4)
50 lb. Machine
200 lb. Foundation
Portsmouth Mounting



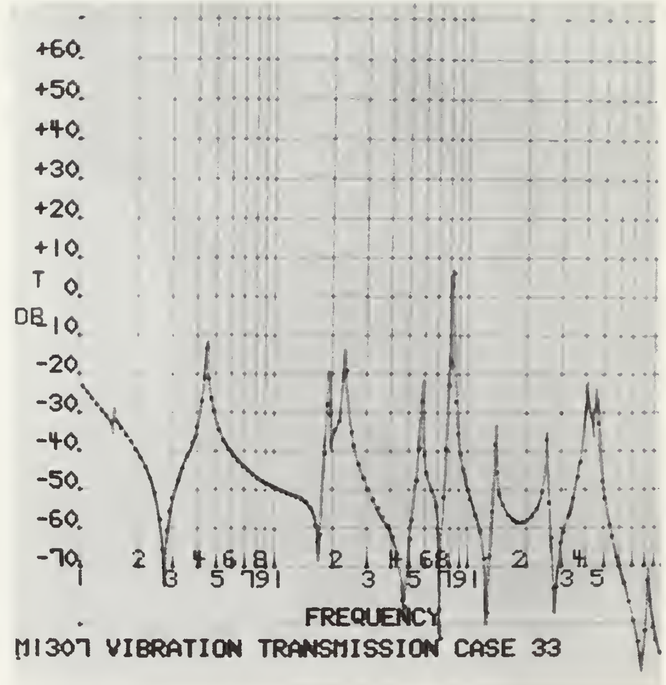
CASE 31

Same as Case 30, except
Decreased Mount Stiffness



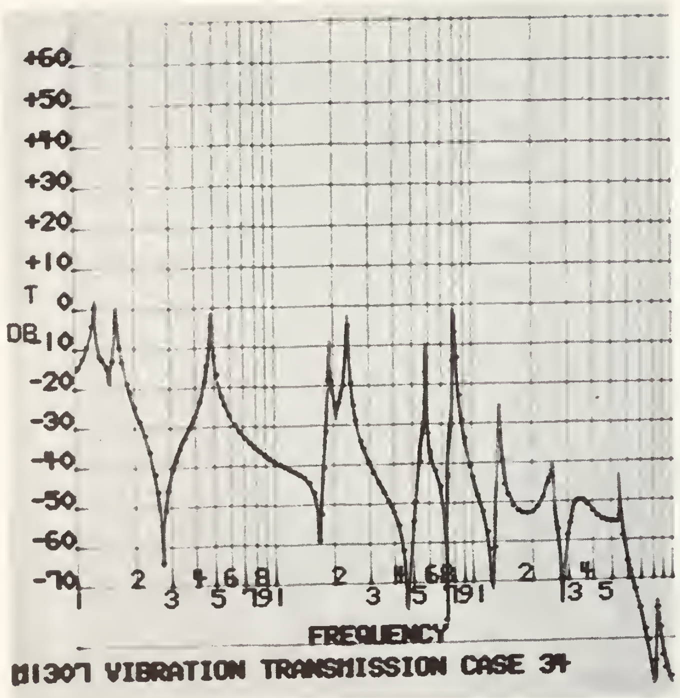
CASE 32

Same as Case 30, except
Thiokol R. D. Mount Replaced
Portsmouth Mount



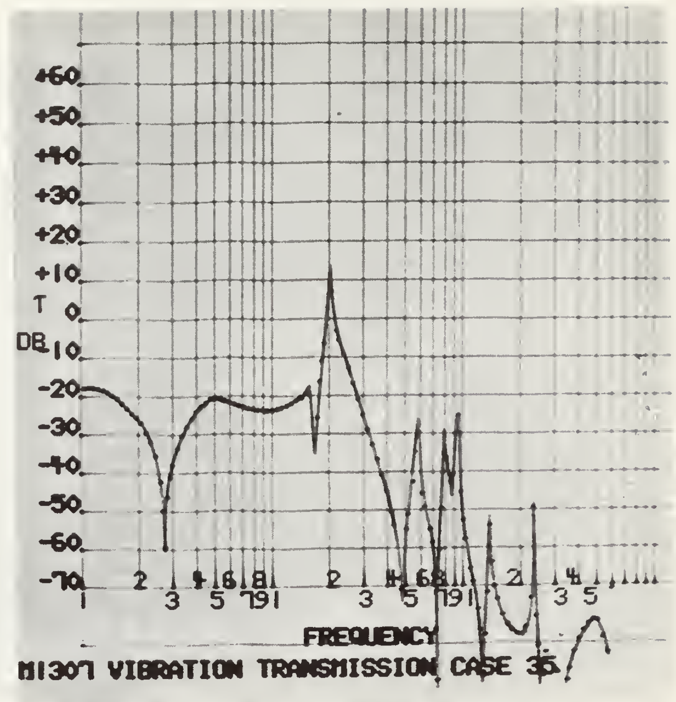
CASE 33

Same as Case 30, except
Increased Mount Damping
Decreased Mount Stiffness
Increased Hull Damping, 20 times



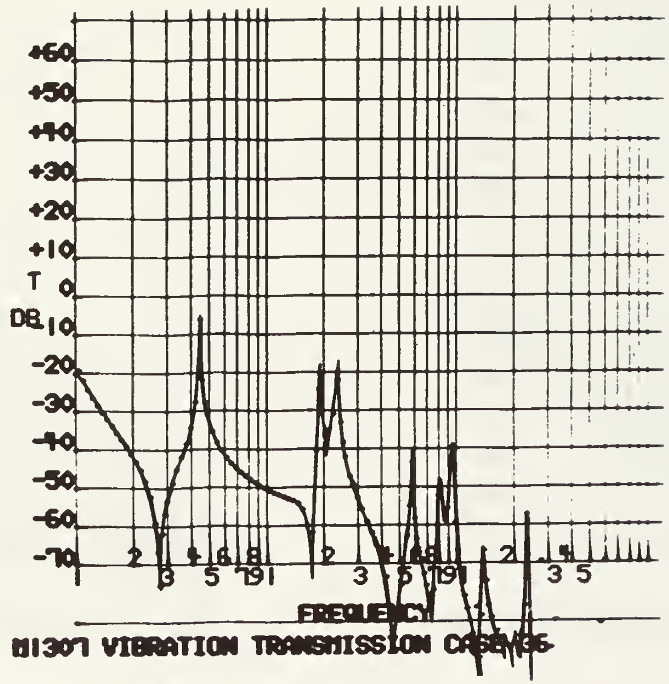
CASE 34

Same as Case 30, except
Foundation Damping, Viscous



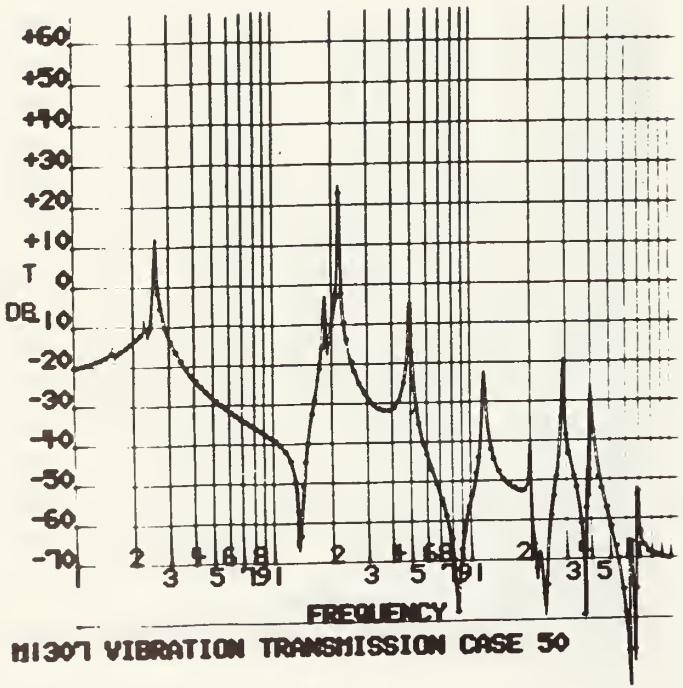
CASE 35

Same as Case 30, except
Thiokol R. D. Mount
Replaced Portsmouth Mount
Foundation Damping, Thiokol R. D.



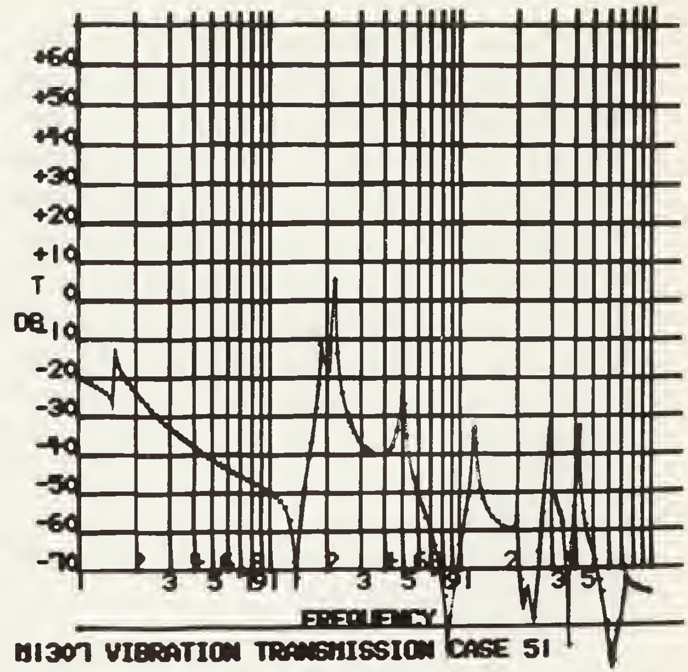
CASE 36

Same as Case 30, except
Foundation Damping, Thiokol R. D.
Decreased Mount Stiffness



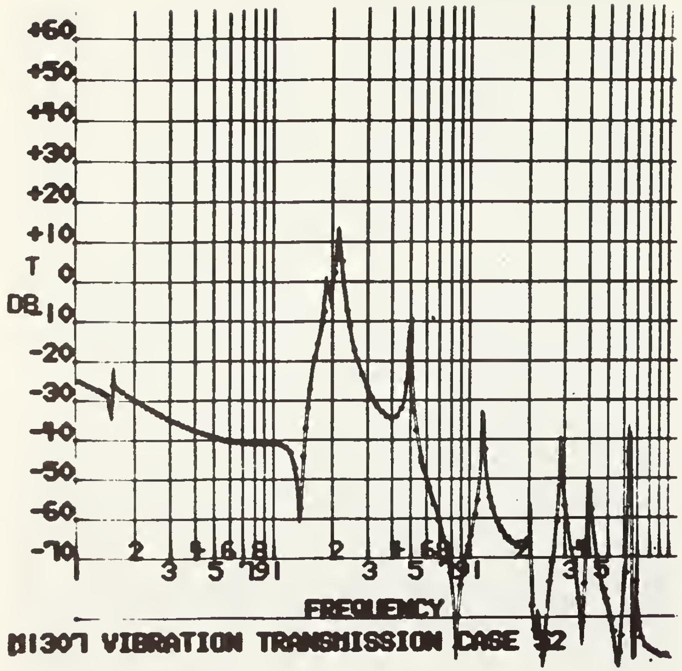
CASE 50

System 4 (Appendix A-5.5)
500 lb. Machine
300 lb. Foundation
Portsmouth Mounting



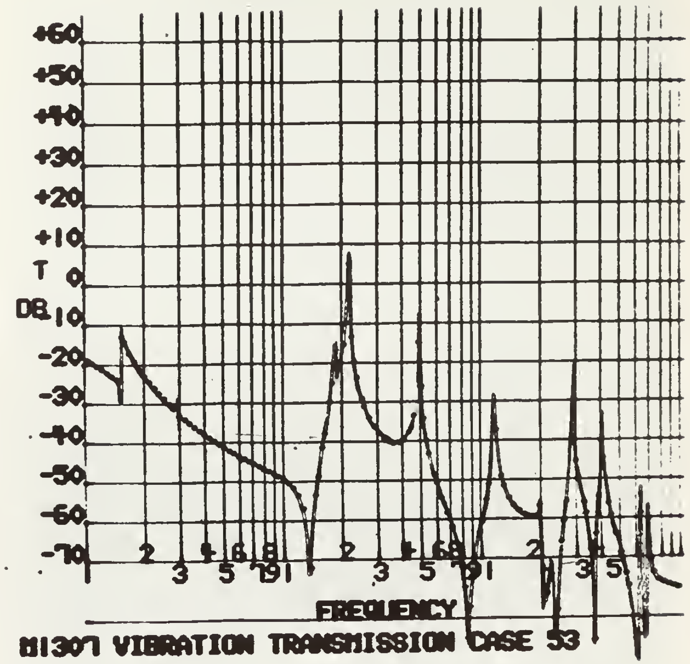
CASE 51

Same as Case 50, except
Decreased Mount Stiffness



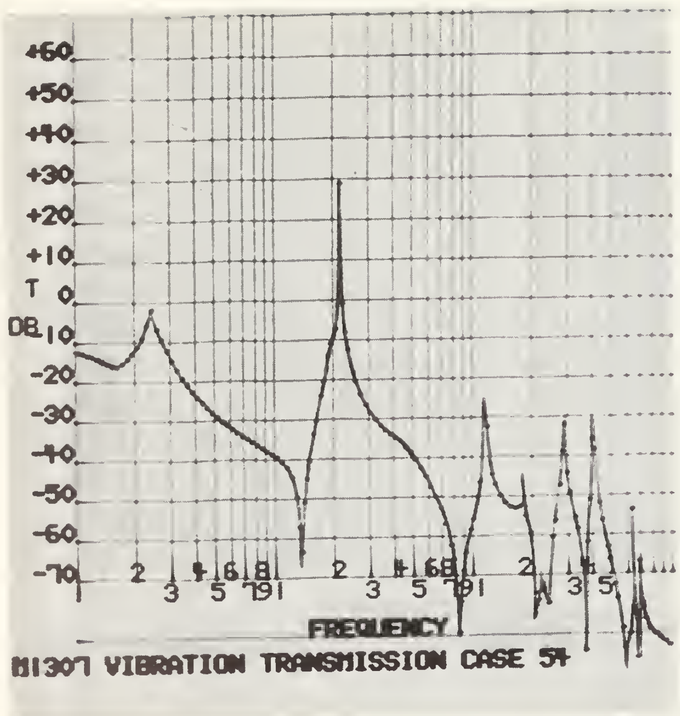
CASE 52

Same as Case 50, except
Thiokol R. D. Mount Replaced
Portsmouth Mount



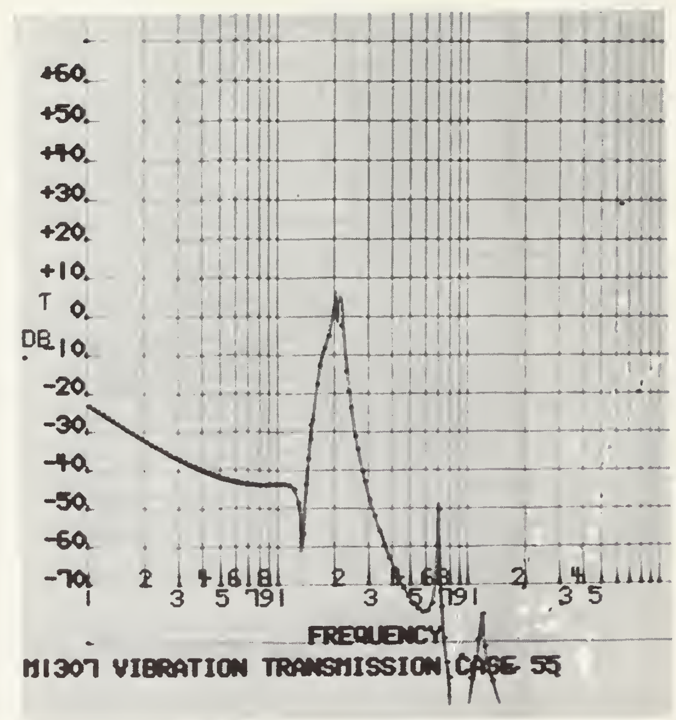
CASE 53

Same as Case 50, except
Increased Mount Damping
Decreased Mount Stiffness
Increased Hull Damping, 20 times



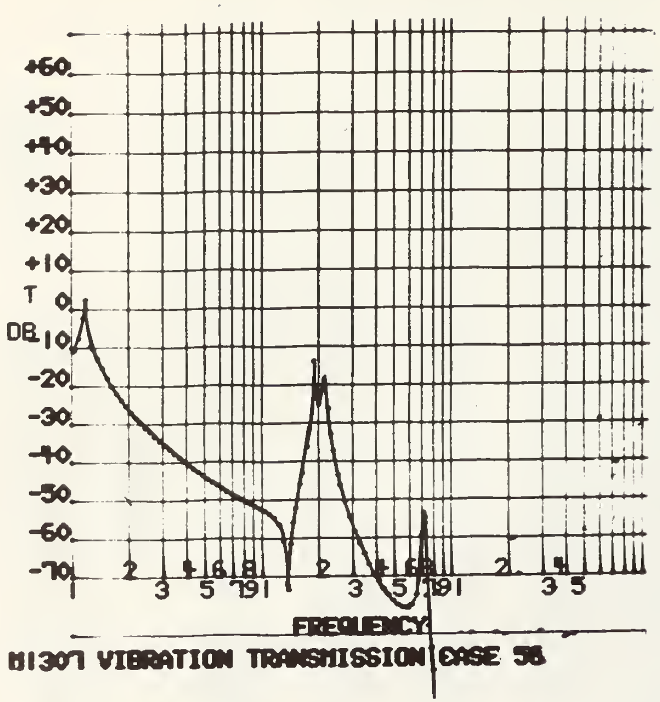
CASE 54

Same as Case 50, except
Foundation Damping, Viscous



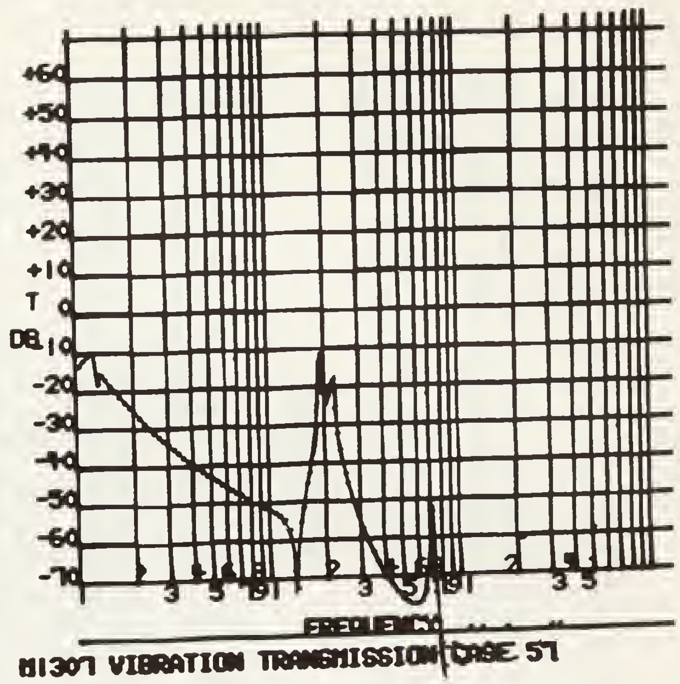
CASE 55

Same as Case 50, except
Thiokol R. D. Mount Replaced
Portsmouth Mount
Foundation Damping, Thiokol R. D.



CASE 56

Same as Case 50, except
Foundation Damping Thiokol R. D.
Decreased Mount Stiffness



CASE 57

Same as Case 50, except
Foundation Damping, Thiokol R. D.
Decreased Mount Stiffness
Increased Mount Damping

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